4

DYNAMICS

NEWTON'S SECOND LAW OF MOTION

(*College Physics* 9th ed. pages 89–92/10th ed. pages 91–94)

Up to this point we have looked at bodies either at rest or traveling with constant velocity. Such bodies have a resultant force of zero acting on them. What happens when the resultant force is not zero? The answer to this question is contained in Newton's second law, which states that *when the resultant force is not zero the body moves with accelerated motion, and that the acceleration, with a given force, depends on a property of the body known as its mass.*

As we have shown in Chapter 2, the *mass* of a body can be considered to be a measure of the quantity of material that makes up the body. Mass is also considered to be that which causes resistance to change of motion, *inertia*. Mass is a scalar. Mass is measured on a balance.

The part of mechanics that includes the study of motion and the forces that cause the motion is called *dynamics*. Equilibrium treats special cases where the acceleration is zero, and kinematics deals with motion only and not its cause, but dynamics is the study of why motion occurs. In its broadest sense, dynamics includes nearly the whole of mechanics.

We know from experience that a body at rest will never start to move of itself; some other body must exert a push or a pull on it. We also know that a force is required to slow or stop a body that is in motion, and that a sidewise or lateral force must be applied to a body to deviate it from its motion along a straight line. Speeding up, slowing down, or changing direction involve a change in either the magnitude or the direction of the velocity of the body. Every time a body is accelerated an external resultant or unbalanced net force must act on it to produce the acceleration.

Newton's second law takes the equation form $\Sigma\vec{F} = m\vec{a}$. It is a simple equation that states that resultant forces always make a "*ma*".

SAMPLE PROBLEM 1

A 12.0 kg mass is to be accelerated at 4.0 m/s^2. What resultant force causes the acceleration?

SOLUTION TO PROBLEM 1

When systems accelerate, there is a resultant or net or unbalanced force causing the acceleration. By Newton's second law,

$$\Sigma F = ma = (12.0\text{ kg})\left(4.0\ \text{m/s}^2\right) = \mathbf{48.0\ N}$$

SAMPLE PROBLEM 2

A mass M is subjected to an unbalanced force of 20.0 N causing it to accelerate from rest to a velocity of 24.0 m/s over a displacement of 10.0 m. Calculate M.

SOLUTION TO PROBLEM 2

The unbalanced force acting on M is $\Sigma F = Ma$ which we can write as $F = Ma$ with the understanding that F also represents the unbalanced force. Since we do not know the time interval involved during the acceleration process, we make use of the time independent equation $v^2 = v_0^2 + 2as$. Since the body starts from rest, the acceleration is $a = \dfrac{v^2}{2s}$. Substituting into $F = Ma$ and solving for M yields $M = \dfrac{2Fs}{v^2}$ and then

$$M = \frac{2(20.0\text{ N})(10.0\text{ m})}{\left(24.0\ \text{m/s}\right)^2} = \mathbf{0.69\ kg}$$

SAMPLE PROBLEM 3

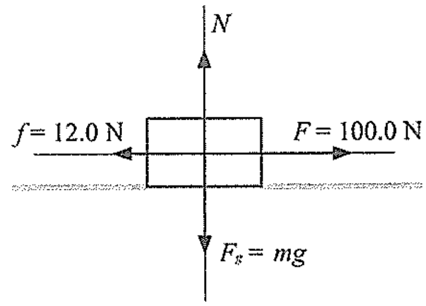
An 80.0 kg wooden block is pushed to the right across a horizontal surface with a force of 100.0 N. The frictional force between the block and the surface is 12.0 N.

(a) What acceleration does the block experience?

(b) Find the coefficient of sliding friction for the surfaces involved.

Solution to Problem 3

First we make a free-body diagram.



(a) Friction always opposes the motion and is negative. The resultant force, ΣF, is $\Sigma F = F - f = ma$ or $ma = F - f$ and solving for a gives

$$a = \frac{F - f}{m} = \frac{(100.0\text{ N} - 12.0\text{ N})}{80\text{ kg}} = \mathbf{1.1\ m/s^2}$$

(b) To find the coefficient of friction we need the normal force. Vertically, the block is in equilibrium in this problem meaning that all of the upward forces equal all of the downward forces, or

$$N = F_g = mg = (80.0\text{ kg})\left(9.8\ \text{m/s}^2\right) = 784\text{ N}$$

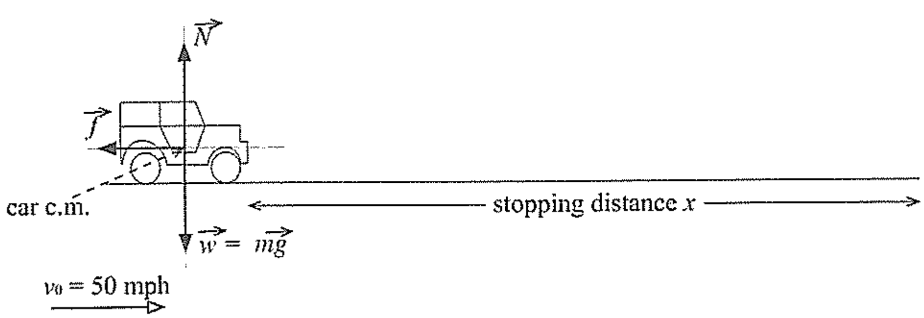
By definition the frictional force is $f = \mu N$ and solving for the coefficient gives

$$\mu = \frac{f}{N} = \frac{12.0\text{ N}}{78.4\text{ N}} = \mathbf{0.015}$$

Recall that coefficients of friction are dimensionless.

Sample Problem 4

A sports vehicle travels at 50.0 mph on a horizontal, dry section of asphalt highway. If the coefficient of friction between the tires and the surface of the highway is 0.88, what is the minimum stopping distance of the car when the brakes are fully applied? Ignore air resistance.



SOLUTION TO PROBLEM 4

First, note that the initial speed of the vehicle is expressed in English units: 50.0 mph. Convert to SI:

$$50.0\frac{\text{mi}}{\text{h}}\times\frac{88\ \text{ft/s}}{60\ \text{mi/h}}\times\frac{1\ \text{m}}{3.28\ \text{ft}}=22.4\ \text{m/s}$$

The vehicle is in equilibrium vertically and $N = w = mg$. Horizontally, the only force acting is the force of friction, $f = \mu N = \mu(mg) = -\mu mg$. The frictional force is an unbalanced force and applying Newton's second law: $f = ma = -\mu mg$ and mass divides out on both sides leaving the acceleration as $a = -\mu g$. Acceleration is negative since the vehicle is slowing and the brakes act in the opposite direction to the motion.

The time element is not given in the problem. We know the initial speed, the final speed, and now the acceleration. We require the time independent equation to determine the stopping distance, x, $v^2 = v_0^2 + 2ax$. Since $v = 0$,

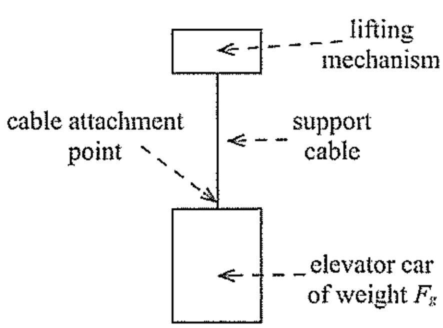
$$0 = v_0^2 + 2(-\mu g)x \text{ and } x = \frac{v_0^2}{2\mu g} = \frac{\left(22.4\ \text{m/s}\right)^2}{2(0.88)\left(9.8\ \text{m/s}^2\right)} = \mathbf{29.1\ m}$$

Stopping distance is related to the square of the initial speed. The faster a vehicle travels, the greater the distance the vehicle requires to stop. Note that we did not need the mass of the vehicle to find the stopping distance.

ELEVATORS

(*College Physics* 9th ed. pages 103–104/10th ed. pages 106–107)

The basic elevator consists of a mechanism for lifting or lowering, a support cable, and an elevator car. The attachment point for the support cable acts as a knot when making vector or free-body diagrams.



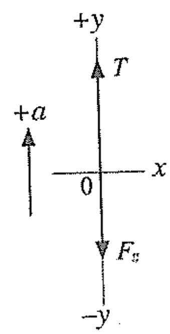
Elevator cars either are being pulled upward or are being lowered by the mechanism and the support cable. Since the car is confined to vertical motion, horizontal motion is not involved in problem work.

SAMPLE PROBLEM 5

Consider an elevator car having a mass of 2000 kg and being accelerated upward at 1.2 m/s^2. What tension exists in the support cable?

SOLUTION TO PROBLEM 5

First make a free-body diagram.



Since the car is accelerating upward, we show the a vector pointing upward and $T > F_g$ in the diagram. Next we write $\Sigma F = T - F_g = ma$. The symbol sigma Σ means to sum but F_g is directed along the −y-axis making it negative. Solving for the tension

$$\begin{aligned} T &= ma + mg \\ &= m(a+g) = (2000\ \text{kg})\left(1.2\ \text{m/s}^2 + 9.8\ \text{m/s}^2\right) \\ &= \mathbf{2.2 \times 10^4\ N} \end{aligned}$$

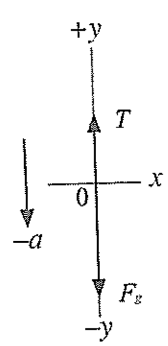
Accelerating upward requires that the tension vector is greater than the weight, $T > F_g$.

SAMPLE PROBLEM 6

Consider an elevator car having a mass of 2000 kg and is being accelerated downward at 1.2 m/s^2. What tension exists in the support cable?

SOLUTION TO PROBLEM 6

First make a free-body diagram.



Since the car is accelerating upward, we show the a vector pointing downward and $T < F_g$ in the diagram. Next we write $\Sigma F = T - F_g = m(-a)$. F_g is directed along the $-y$-axis making it negative. The acceleration vector is also directed downward making it negative in this case. Solving for the tension

$$\begin{aligned} T &= mg - mg \\ &= m(g - a) \\ &= (2000 \text{ kg})\left(9.8 \ {}^{\text{m}}\!/\!_{\text{s}^2} - 1.2 \ {}^{\text{m}}\!/\!_{\text{s}^2}\right) \\ &= \mathbf{1.7 \times 10^4 \text{ N}} \end{aligned}$$

Accelerating downward requires that the weight vector is greater than the tension, $T < F_g$.

Sample Problem 7

What is the maximum downward acceleration an elevator car can have?

Solution to Problem 7

The maximum acceleration is the acceleration due to gravity, g. This could only happen if the support cable were detached from the elevator car placing it in free-fall.

Sample Problem 8

What is the tension in the support cable if the 2000.0 kg elevator car is

(a) at rest in the elevator shaft?

(b) moving upward at constant speed?

(c) moving downward at constant speed?

Solution to Problem 8

In all three cases, the elevator car is NOT accelerating. It is in a state of equilibrium. In such a state, all the upward forces always equal all the downward forces.

$$T = F_g = mg = (2000.0 \text{ kg})\left(9.8 \ {}^{\text{m}}\!/\!_{\text{s}^2}\right) = \mathbf{1.96 \times 10^4 \text{ N}}$$

AP Tip

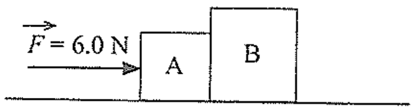
When encountering a problem dealing with forces, ask the question: "Is the system accelerating?" If the system accelerates Newton's second law is involved. If there is no acceleration, the system is in equilibrium.

Two–Body Systems

(*College Physics* 9th ed. pages 104, 109–110/10th ed. pages 107, 112–113)

Sample Problem 9

Two wooden blocks, A and B, with masses of $m_A = 4.0\ \text{N}$ and $m_B = 6.0\ \text{kg}$ respectively, are in contact on a frictionless surface. If a horizontal force of $F = 6.0\ \text{N}$ pushes them to the right, what force does m_A exert on m_B?



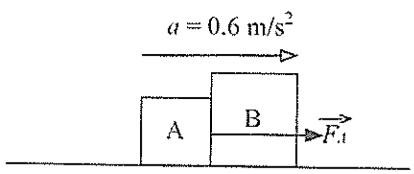
Solution to Problem 9

Blocks A and B act as a single system of mass $M = m_A + m_B = 4.0\ \text{kg} + 6.0\ \text{kg} = 10.0\ \text{kg}$. An unbalanced force $F = 6.0\ \text{N}$ is exerted on the system giving it acceleration

$$a = \frac{F}{M} = \frac{(6.0\ \text{N})}{(10.0\ \text{kg})} = 0.6\ \text{m}/\text{s}^2 .$$

Keep in mind that the entire system accelerates at this rate.

Block A pushes into block B with a force F_A.



By Newton's second law,

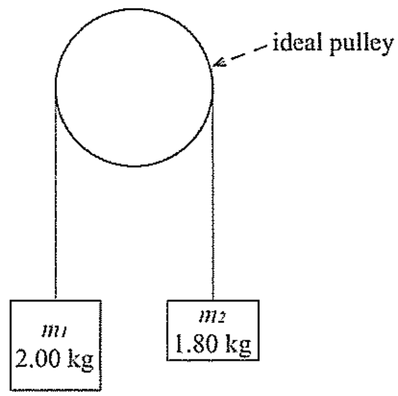
$$F_A = m_B a = (6.0\ \text{kg})\left(0.6\ \text{m}/\text{s}^2\right) = \mathbf{3.6\ N}$$

Note that since F_A is perpendicular to the surface of block B, the force B exerts on A is a normal force, *N*.

Atwood's machine is a physics laboratory device that is used to experimentally study several areas of physics. One of these areas is dynamics, Newton's second law of motion. The Atwood machine consists of a pulley, an ideal pulley in this case, a cord that passes over the pulley and two masses attached one at each end. The cord is considered massless and ideally does not stretch. Recall that an ideal pulley is one that is frictionless and whose mass and radius are not factors to consider.

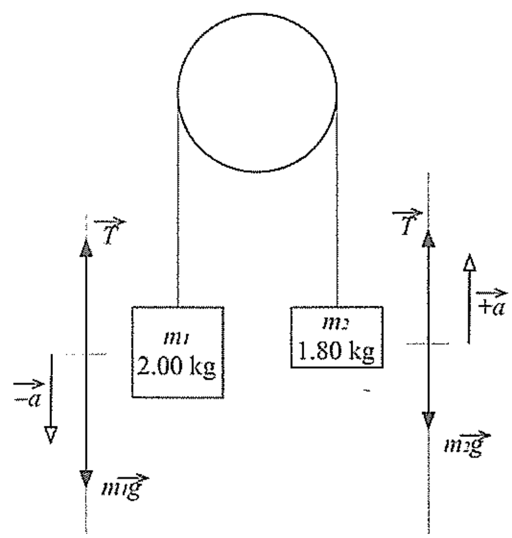
SAMPLE PROBLEM 10

Consider the drawing given below of an Atwood's machine. The masses are held at rest and then are released. The mass m_1 falls and m_2 rises. Calculate the acceleration experienced by the system and the tension in the cord.



SOLUTION TO PROBLEM 10

First, make a free-body diagram. Since m_1 falls its acceleration is negative. The mass m_2 rises making its acceleration positive.



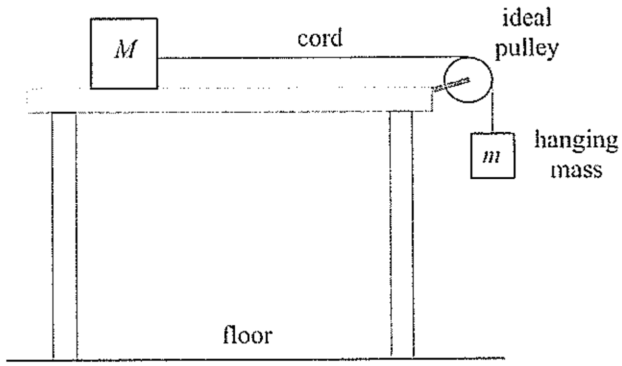
Applying Newton's second law to m_1, $\Sigma F = T - w_1 = T - m_1 g = m_1(-a)$. Solving for T in terms of a, $T = m_1 g - m_1 a$.

Next we apply Newton's second law to m_2, $\Sigma F = T - w_2 = T - m_2 g = m_2 a$. Again solving for T in terms of a, $T = m_2 a + m_2 g$. Since $T = T$, we can write $m_2 a + m_2 g = m_1 g - m_1 a$. Transposing yields $m_1 a + m_2 a = m_1 g - m_2 g$. Factoring: $(m_1 + m_2)a = (m_1 - m_2)g$ and

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g$$
$$= \frac{(2.00\text{ kg} - 1.80\text{ kg})}{(2.00\text{ kg} + 1.80\text{ kg})}\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)$$
$$= \left(\frac{0.20\text{ kg}}{3.80\text{ kg}}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)$$
$$= \mathbf{0.52\ m/s^2}$$

From above,

$$T = m_2 a + m_2 g$$
$$= m_2(a + g) = (1.80\text{ kg})\left(0.52\ \text{m/s}^2 + 9.8\ \text{m/s}^2\right)$$
$$= \mathbf{18.6\ N}$$

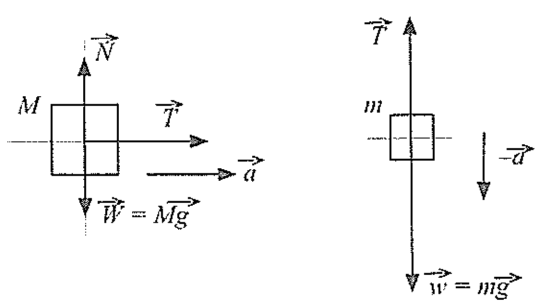


SAMPLE PROBLEM 11

Consider the system shown above. The system is being held in place. Ignoring friction and the mass of the connecting cord, if block M has a mass of 5.0 kg and block m has mass of 12.0 kg, what acceleration will the masses experience when released from rest and what tension will exist in the cord connecting them?

SOLUTION TO PROBLEM 11

First make a free-body diagram for both masses.



Block M is in equilibrium vertically; however, it is NOT in equilibrium horizontally. It will accelerate to the right with acceleration a.

Since the system is connected with a cord and we are dealing with an ideal pulley, the entire system experiences the same acceleration. The tension in the cord at every point will have the same magnitude. The ideal pulley changes the direction of the acceleration and tension vectors, nothing else.

For block M apply Newton's second law, $\Sigma F = T = Ma$ and for block m, $\Sigma F = T - w = T - mg = ma$ and $T = mg + m(-a) = m(g - a)$.

Since $T = T$, then $Ma = m(g - a) = mg - ma$. Transposing and factoring gives $Ma + ma = a(M + m) = mg$. Solving for the acceleration yields $a = \frac{m}{M+m}g$.

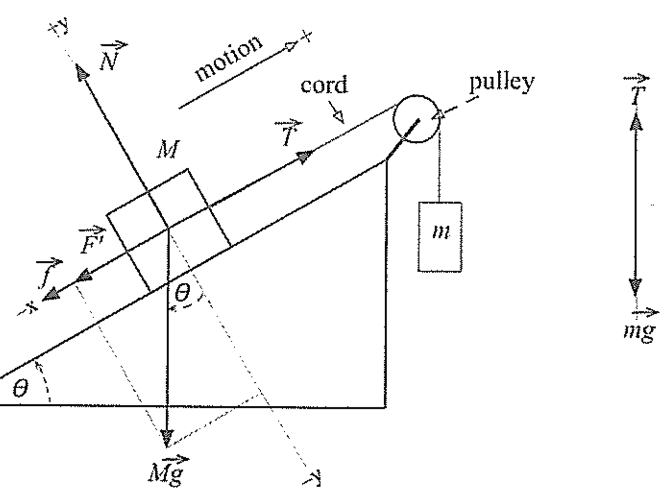
$$a = \frac{m}{M+m}g = \frac{12.0\text{ kg}}{5.0\text{ kg} + 12.0\text{ kg}}\left(9.8\ \text{m/s}^2\right) = \mathbf{6.9\ m/s^2}$$

and

$$T = Ma = (5.0\text{ kg})\left(6.9\ \text{m/s}^2\right) = \mathbf{34.6\ N}$$

Sample Problem 12

An inclined plane is angled at 25° and an ideal pulley is attached to the upper end. A block of mass $M = 12.0$ kg is held at rest on the surface of the plane and is attached to a cord that runs over the pulley to a mass m. The coefficient of friction for the contact surfaces on the inclined plane is $\mu = 0.40$. When released, mass M accelerated up the plane at $a = 1.20\ \text{m/s}^2$. What is mass m? Ignore any stretching effects in the cord and its mass.



Solution to Problem 12

The system is not in equilibrium and all the forces up the plane *DO NOT* equal all the forces down the plane. The force up the plane is greater than the sum of the forces down the plane. Recall that the ideal pulley serves to change the direction of the tension T vector.

Since the block M accelerates up the plane, we apply Newton's second law and write

$$\Sigma F = T - F' - f = Ma \text{ and } T - Mg\sin\theta - \mu Mg\cos\theta = Ma$$

To find the mass at the other end of the string we need the tension in the cord, so we solve for T.

$$T = Mg\sin\theta + \mu Mg\cos\theta + Ma$$

simplifying:

$$T = M(g\sin\theta + \mu g\cos\theta + a)$$

$$T = 12.0\text{ kg}\left(9.8\ {}^{\text{m}}\!/_{\text{s}^2} \times \sin 25° + 0.40 \times 9.8\ {}^{\text{m}}\!/_{\text{s}^2} \times \cos 25° + 1.20\ {}^{\text{m}}\!/_{\text{s}^2}\right)$$
$$= 106.7\text{ N}$$

Mass m is falling and we can write,

$$T - mg = ma$$

Solving for m,

$$m = \frac{T}{(g-a)} = \frac{(106.7\text{ N})}{\left(9.8\ {}^{\text{m}}\!/_{\text{s}^2} - 1.2\ {}^{\text{m}}\!/_{\text{s}^2}\right)} = \mathbf{12.4\ kg}$$

Dynamics: Student Objectives for the AP Exam

- You should be able to differentiate between systems in equilibrium and systems not in equilibrium.
- You should be able to explain how a particle can move if no net force is acting on it.
- You should be able to state Newton's second law of motion.
- You should be able to make and discuss free-body diagrams and to use them in problem solutions.
- You should be able to explain the reason for the direction of the force of friction.
- You should be able to define what is meant by an ideal pulley.

Multiple-Choice Questions

1. A 5.00 kg body is supported by a single cord that hangs from the ceiling of a room. The tension in the cord is
   (A) 5.00 kg
   (B) 5.00 N
   (C) 49.0 kg
   (D) 49.0 N