2

EQUILIBRIUM

MASS AND INERTIA

(*College Physics* 9th ed. pages 89–91/10th ed. page 91)

Mass is an *intrinsic property* shared by all bodies. It is a measure of the quantity of material present in a body. In our everyday world, the mass of a body is considered to be constant. Mass *m* is a scalar quantity and its SI unit is the kilogram, kg.

Mass is also a measure of the *inertia* of a body and is not in any way dependent upon gravity. Inertia can be defined as the resistance to any change in the motion of a body. Bodies of small mass have a small inertia and can be moved about rather easily. Very large bodies have a very large inertia and are difficult to move about.

Picture a large truck, a tractor-trailer, having a mass of 18,000 kilograms at rest in neutral with the parking brake released. It sits in a large, empty, flat parking lot. The truck, if weighed at a truck scale would weigh about 40,000 pounds. This truck has quite a large inertia. We have twenty very strong football players get behind the truck and push in unison. Slowly, the truck begins to move. To make it move, the force all the football players simultaneously exert on it overcomes its inertia and now it moves on its own across the parking lot at five miles an hour. Its cab is locked. It is a large moving truck with a large inertia. The question is how do we make it stop? Its inertia must be overcome to bring it to rest.

WEIGHT

(*College Physics* 9th ed. pages 92–93/10th ed. pages 95–96)

Weight and mass are *NOT* the same thing. Weight, $\vec{w}$, is the force of gravitational attraction on a body and is solely dependent upon the strength of a gravitational field. Weight is a vector and its direction is always vertically downward toward the center of the Earth. Think of the center of the Earth as a tiny black hole where all of its mass is concentrated. The weight of a body then is the gravitational force with which the body is pulled toward the center of the Earth.

Mass has another important property. Masses generate *gravitational fields*, $\vec{g}$-fields. Small masses have a small $\vec{g}$-field associated with them and very large masses generate very large gravitational fields. The reason is not fully understood. Compared to you or me, the $\vec{g}$-field of the Earth is huge and extends out a great distance into space. It is the interaction between the $\vec{g}$-fields of the Earth and the moon that brings about the gravitational attraction between them.

The weight of a body on the surface of the Earth is the gravitational attraction the Earth and the body exert on one another. If the body weighs 100 N it means that the gravitational attraction between the body and the Earth is 100 N. We have stated that in our everyday world that mass is a constant. Weight, however, is not.

Assuming that the Earth is homogeneous and is a sphere, at its surface, the $\vec{g}$-field of the Earth has a constant value. For the most part it is constant. The *strength* or *intensity* of the $\vec{g}$-field of the Earth at or within a few kilometers of the surface is

$$\vec{g} = -9.8\ \mathrm{m/s^2}$$

The intensity, $\vec{g}$, is a vector and is directed vertically downward toward the center of the Earth. This intensity weakens as you move further from the center of the Earth. In dealing with falling bodies and projectile motion, the intensity is also called the acceleration due to gravity and, within a few kilometers of the surface, has the value

$$\vec{g} = -9.8\ \mathrm{m/s^2}$$

Weight, W or w, or the force of gravity, F_g may be defined as the product of the mass of a body and the acceleration due to gravity. Weight is a vector, mass is a scalar, and $\vec{g}$ is a vector.

$$\vec{W} = \vec{w} = \vec{F}_g = m\vec{g}$$

Knowing that weight is a vector and is always directed vertically downward toward the center of the Earth we can write

$$w = mg$$

SAMPLE PROBLEM 1

Determine the weight of a 75.0 kg body at rest on the surface of the Earth.

Solution to Problem 1

Weight is defined as $w = mg = (75.0\ \text{kg})\left(9.8\ {}^{\text{m}}\!/_{\text{s}^2}\right) = \mathbf{735.0\ N}$

Note the product $(1\ \text{kg})\left(1\ {}^{\text{m}}\!/_{\text{s}^2}\right) = 1\ \text{kg}\cdot{}^{\text{m}}\!/_{\text{s}^2} = 1\ \text{N}$

> **AP Tip**
>
> Weight is a vector and is expressed in N. Weight is measured on scales and is dependent on location.

Sample Problem 2

A small pickup truck has a weight of 14.7×10^3 N. What is the mass of the truck?

Solution to Problem 2

The mass of a body is related to weight at or near the surface of the Earth as

$$m = \frac{w}{g} = \frac{\left(14.7\times10^3\ \text{N}\right)}{9.8\ {}^{\text{m}}\!/_{\text{s}^2}} = \frac{14.7\times10^3\ \text{kg}\cdot{}^{\text{m}}\!/_{\text{s}^2}}{9.8\ {}^{\text{m}}\!/_{\text{s}^2}} = \mathbf{1.5\times10^3\ kg}$$

By definition, 1.0 metric ton = 1.0 MT = 1,000 kg.

We could also have expressed the answer as

$$1.5\times10^3\ \text{kg}\times\frac{1.0\ \text{MT}}{1.0\times10^3\ \text{kg}} = \mathbf{1.5\ MT}$$

> **AP Tip**
>
> Mass is a scalar and is expressed in kg. Mass is measured on a balance. Mass in our everyday experience is constant and is independent of location.

Sample Problem 3

If the pickup truck of Problem 2 were placed on the surface of the planet Mars, and knowing that the gravitational intensity for the surface of Mars is $\vec{g}_{Mars} = 3.7\ {}^{\text{m}}\!/_{\text{s}^2}$, what is the weight of the truck on the surface of Mars?

Solution to Problem 3

The weight of the pickup truck on the planet Mars is

$$w_{Mars} = mg_{Mars} = \left(1.5\times10^3\ \text{kg}\right)\left(3.7\ {}^{\text{m}}\!/_{\text{s}^2}\right) = \mathbf{5.6\times10^3\ N}$$

Mars is a smaller planet with a smaller $\vec{g}$-field. Weight will vary from place to place whereas mass remains constant.

CENTER OF MASS

(*College Physics* 9th ed. pages 241–243/10th ed. pages 246–249)

As we have seen in the first chapter, *simultaneous forces* are forces that are applied to a body at the same time and *concurrent forces* are forces whose lines of action *all* pass through a common point. All regular shaped bodies have a unique point within the body that we call the *center of mass*. We will abbreviate center of mass as c.m. for the remainder of the text. Real bodies all behave as if all their mass is concentrated at the center of mass. Since the Earth behaves as if all its mass were concentrated at its c.m., the $\vec{g}$ -field of the Earth appears to "sink" into its center.

Each and every particle on the Earth has a force acting on it that is common with every other particle and that is its weight. Think of any extended body as being made up of many particles each with a weight vector that points vertically downward to the center of the Earth. It makes no difference as to the shape or size of a body, there exists a unique point where the entire weight may be considered to be concentrated. This point is called the *center of gravity*, c.g., of the body. The c.g. of a regular shaped body such as a cube, uniform sphere, rod, or meterstick is located at the geometrical center of the body. Irregular shaped bodies have centers-of-gravity that may be located outside the material of the body.

The c.m. is identical to the c.g. and the two terms are often used interchangeably. However, the c.m. is preferable in the present context because gravity is not involved. Regular shaped bodies like cubes, spheres, solid disks, and blocks of wood or stone that are homogeneous will have their c.m. at their geometric center. Finding the c.m. of irregular shaped bodies and systems of bodies is a bit complex and will not be considered at this time.

AP Tip

If a single force is applied to the c.m. of a body, purely straight-line motion will occur.

AP Tip

A body is any object and any object is a body. A particle is a tiny body, so small in fact that we can treat it as a point. Under certain conditions we can treat a body as large as the sun as a particle as well as treating a baseball as one, too.

FORCE

(*College Physics* 9th ed. pages 87–88/10th ed. pages 89–90)

In the previous chapter we defined a force as a push or a pull on a body. We also saw how to add up the forces acting on a body: *forces always add as vectors.*

In the world around us, we encounter two classes of force, *contact forces* and *field forces*. In the case of contact forces, the body producing the force actually touches the body being acted upon. With field forces, the force is experienced over a distance such as observed in the case of the force of gravity of the Earth acting on a satellite in Earth orbit.

Gravity is one of the four fundamental forces in nature, and it is the weakest of the four. Gravity is the primary force acting on all astronomical bodies. It is always an attractive force. The force of gravity between two ordinary bodies on the Earth is negligibly small.

On the atomic level physics recognizes that there are four fundamental forces: (1) gravitational, (2) electromagnetic, (3) weak nuclear, and (4) strong nuclear. The contact forces we experience in daily life only appear to be contact forces. Actually, most contact forces are the electromagnetic forces acting as a field force at very small distances.

AP Tip

It is a common practice to treat bodies as a point located at the c.m. of the body and to attach the origin of a frame of reference there.

NEWTON'S FIRST LAW OF MOTION

(*College Physics* 9th ed. pages 88–89/10th ed. pages 90–91)

The part of physics we call mechanics is based on Newton's three Laws of Motion clearly stated for the first time by Sir Isaac Newton (1642–1727) which were published in 1686 in his *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy).

AP Tip

A major effect of a force is to alter the state of motion of a body.

Newton's first law can be stated as: *A body at rest remains at rest; a body traveling at constant velocity will remain traveling at constant velocity unless an external unbalanced force acts on the body.*

Recall that constant velocity means constant speed in a straight line in a given direction.

We have already seen that the property of a body that allows it to remain at rest or maintain a constant state of motion is called inertia. Newton's first Law is also called the Law of Inertia.

Newton's Third Law of Motion

(*College Physics* 9th ed. pages 95–98/10th ed. pages 97–100)

For every force that acts on a body, there is always a reaction force that is equal in magnitude and is opposite in direction. This is a statement of Newton's third law of Motion.

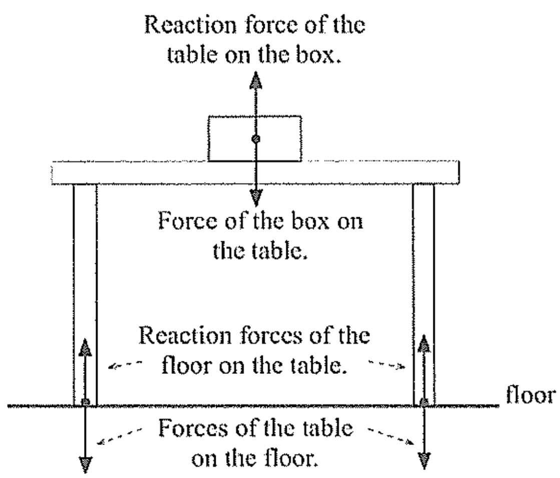
Action-reaction forces act on two different bodies and that means that they do not cancel one another. When two bodies contact one another, the action-reaction forces are called *contact forces*.

AP Tip

There can never be a single isolated force. There can be no force unless two bodies are involved.

Consider the drawing of a table with a box resting on its surface. The box pushes downward on the tabletop. The tabletop pushes vertically upward on the box. The two forces are an action-reaction pair. Their vector sum is zero.

The table pushed vertically downward on the floor and the floor in return pushes vertically upward on the table. The forces are action-reaction pairs and again, the vector sum is zero.



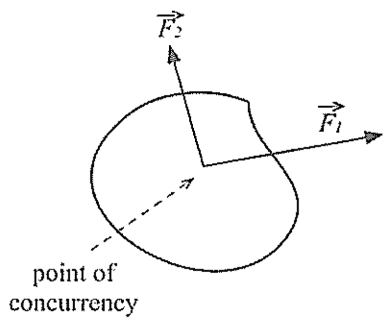
Concurrent Forces and Equilibrium

(*College Physics* 9th ed. pages 99–101/10th ed. pages 101–103)

If a body is at rest when acted upon by simultaneous and concurrent forces, it is in a state of *static equilibrium*. If a body travels in a straight line with constant velocity when acted on by simultaneous and concurrent forces, it is in a state of *translational equilibrium*. There is no acceleration when a body or a system is in a state of equilibrium. In such a system, the resultant force acting is always zero, $\Sigma\vec{F} = \vec{R} = 0$.

SAMPLE PROBLEM 4

Consider the body shown below. Two concurrent forces, $\vec{F}_1 = 20.0$ N at 10.0° and $\vec{F}_2 = 15.0$ N at 110.0°, act on the body.

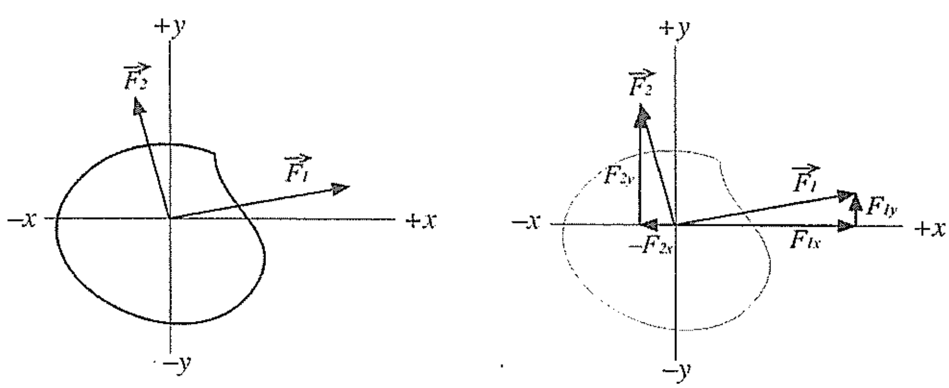


(a) By inspection, determine the resultant force, $\vec{R}$, the body experiences.

(b) For a body to be in a state of equilibrium, $\vec{R} = 0$, we can add a force to the system that will place it into a state of equilibrium. We call such a force the *equilibrant*, $\vec{E}$. What equilibrant will place the system into a state of equilibrium?

SOLUTION TO PROBLEM 4

(a) The first thing we do is attach a frame of reference to the point of concurrency. Then we sketch in and identify the *x*- and *y*-components.

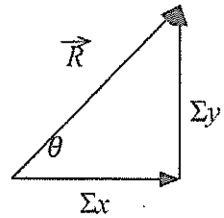


Next, we add the *x*- and *y*-components.

$$\Sigma x = F_1 \cos\theta_1 + F_2 \cos\theta_2 = (20.0 \text{ N})(\cos 10.0°) + (15.0 \text{ N})(\cos 110.0°) = 14.57 \text{ N}$$

$$\Sigma y = F_1 \sin\theta_1 + F_2 \sin\theta_2 = (20.0 \text{ N})(\sin 10°) + (15.0 \text{ N})(\sin 110.0°) = 17.57 \text{ N}$$

Recall that the sums of the components are the x- and y-component of the resultant.



Calculating the magnitude of the resultant:

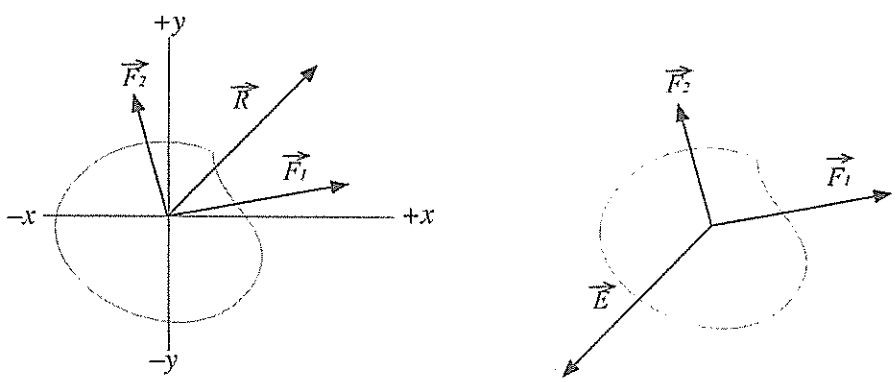
$$R=\sqrt{(\Sigma x)^2+(\Sigma y)^2}=\sqrt{(14.57\text{ N})^2+(17.57\text{ N})^2}=\mathbf{22.8\text{ N}}$$

Finding the angle,

$$\theta=\tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)=\tan^{-1}\left(\frac{17.57\text{ N}}{14.57\text{ N}}\right)=\mathbf{50.3°}$$

The body would move as if a 22.8 N force were pulling it at a 50.3° angle.

(b) The equilibrant E is a force that is equal in magnitude to the resultant but is directed 180° away from the resultant.



Forces that are not in equilibrium may be put into equilibrium by the addition of a force equal and opposite to the resultant. This equal but opposite force to the resultant of a system of forces is the *equilibrant.*

In this problem, the equilibrant then must have a magnitude of $E=22.8\text{ N}$ at an angle of $\theta=50.3°+180°=230.3°$. The equilibrant is $\vec{E}=\mathbf{22.8\text{ N} @ 230.3°}$. The equilibrant "cancels-out" or nullifies the resultant. Now there is *NO* resultant force acting on the body.

When there is no resultant force, all of the x- and y-components in the system must be zero. This is a statement of the *First Condition for Equilibrium,* $\Sigma x=0$ and $\Sigma y=0$. A system is in a state of static equilibrium or in a state of translational equilibrium if and only if the sum of forces acting on the system is zero.

THE NORMAL FORCE

(*College Physics* 9th ed. pages 96–97/10th ed. pages 97–99)

When a body rests on a surface, the surface provides a force on the body in a direction perpendicular to the surface. We call this perpendicular force the *normal force* $\vec{N}$. In this context the word normal means perpendicular.

The origin of the normal force is the interaction between the particles in a solid that act to maintain its shape. Particles are atoms or molecules or ions or some combination of them. Solid-state chemical bonds hold the particles together and are responsible for the normal force.

In most many cases, the normal force is equal to the weight of the body pressing downward on the surface; however, as we will see the normal force can be greater or lesser than the weight of the body.

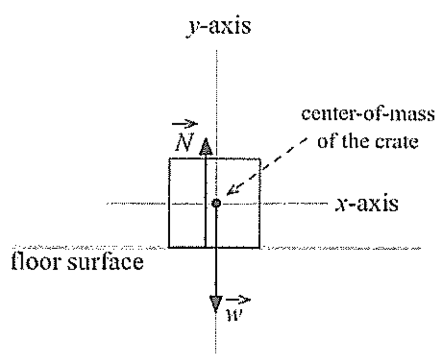
SAMPLE PROBLEM 5

A wooden crate having a weight of 200.0 N sits at rest on the floor of a storeroom. Determine the normal force exerted on the crate.

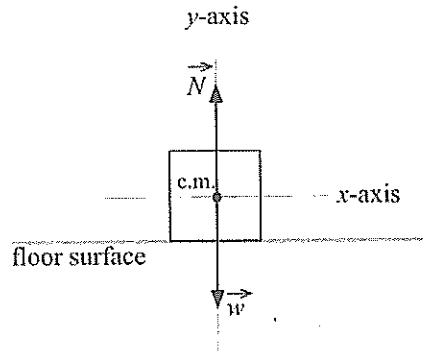
SOLUTION TO PROBLEM 5

First we make a free-body or vector diagram of the problem. We attach a frame of reference to the center of mass of the crate. The crate behaves as if all its mass were concentrated there. The normal force originates directly beneath the c.m. at the interface between the undersurface of the crate and the surface of the floor.

In the free-body diagram, we shifted the normal slightly to the left making it completely visible.



As we pointed out earlier in this chapter as an AP Tip, it is common practice to treat bodies as a point located at the c.m. of the body and to attach the origin of a frame of reference there.

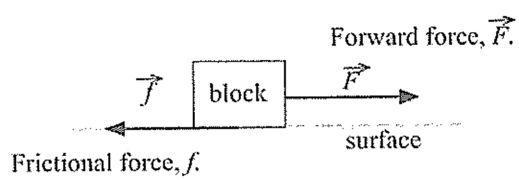


Since the crate is in equilibrium vertically, $\Sigma y = N - w = 0$. Transposing we have $N = w =$ **200.0 N**

FRICTION

(*College Physics* 9th ed. pages 105–110/10th ed. pages 108–114)

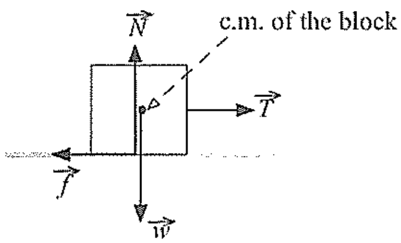
When the surface of a body slides over the surface of another body, each body exerts a *frictional force,* $\vec{f}$, on the other. This frictional force is parallel or tangent to the contact surfaces. The force on each body is opposite to the direction of its motion or tendency toward motion relative to the other. When a block slides from left to right along a laboratory table, a frictional force acts to the left.

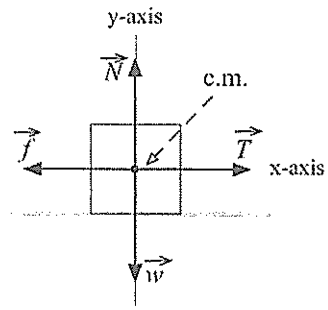


Frictional force may also act when there is no relative motion.

The origin of this type of frictional force is not fully understood. Basically, sliding friction is caused by several factors. Microscopically, surfaces have hills and valleys. The smoother a given surface, the smaller the hills and valleys that cover that surface. As the block slides over the tabletop, the hills and valleys *snag* tending to hold the block back or even prevent it from moving at all. Too, sub-microscopic electrical interactions, *adhesive forces,* between the atoms and molecules of the contact surfaces contribute to the frictional force. The mechanism between the frictional force between a brick and a tabletop and the frictional force between a wooden crate and the surface of a floor will be quite different.

Again, it is common practice to treat bodies as a point located at the c.m. of the body and to attach the origin of a frame of reference there.





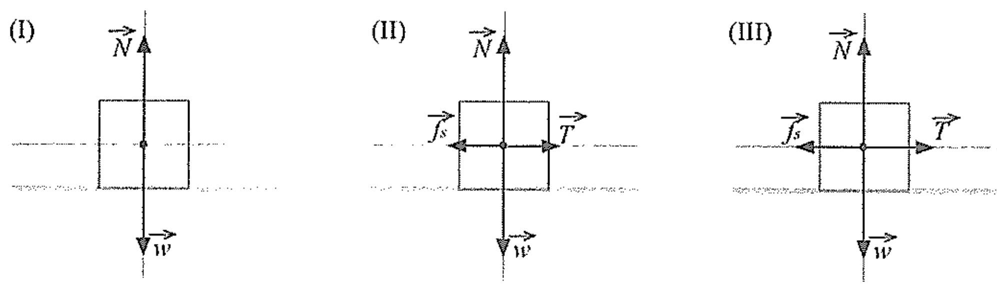
An interesting feature concerning friction forces between contact surfaces is that the frictional force is independent of the contact area. The force of friction depends on two things, one is called the *coefficient of friction*, μ, and the other is the *normal force*, $\vec{N}$, the lower surface exerts on the under surface of the block. We define the maximum value of friction as:

$$\vec{f} = \mu\vec{N}$$

In a state of translational or static equilibrium, $\Sigma\vec{R} = 0$, there is no resultant force. A body in equilibrium is either at rest or moving with constant velocity.

$$\vec{f}_s \leq \mu_s\vec{N}$$

Suppose that a cord is attached to the block in the diagram below and the tension, $\vec{T}$, is gradually increased.



A 100.0 N crate rests on a horizontal floor as in diagram (I) above. The coefficient of static friction between the surfaces is 0.4. What is the maximum static frictional force that can exist between the surfaces? What static frictional force exists?

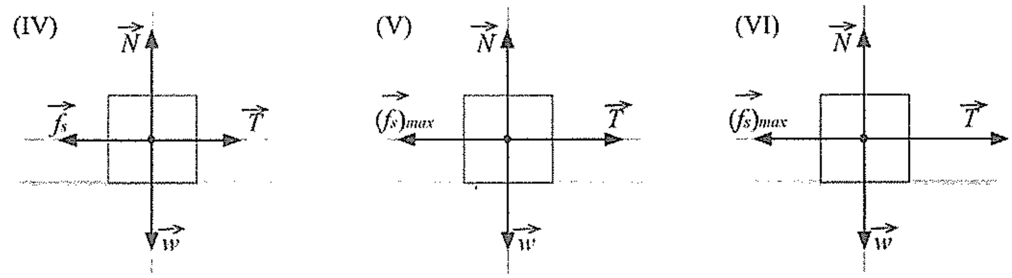
Note that the only forces acting in the system are the weight of the crate and the normal the floor exerts on the crate. The crate is in equilibrium vertically; the upward force equals the downward force. The floor prevents the crate from moving vertically. Therefore $N = w$.

$$(f_s)_{\max} = \mu_s N = (0.4)(100.0\text{ N}) = 40.0\text{ N}$$

Since no horizontal force acts on the crate, no frictional force exists between the crate and floor.

In diagram (II), a rope is attached to the crate and is pulled to the right with a force of $T = 10.0\text{ N}$. What static frictional force exists? Since the maximum static frictional force that will exist between the surfaces is 40.0 N, a force of 10.0 N to the right is not enough to initiate motion. The crate remains in equilibrium and the force to the right equals the force to the left, the static frictional force is 10.0 N.

In diagram (III), the tension in the rope is 20.0 N making the frictional force 20.0 N.



In the above diagram, diagram (IV), the rope is pulled with a force of 30.0 N making the tension in the rope 30.0 N. In equilibrium, all the forces to the right equal the forces to the left and the static frictional force between the surfaces is $T = f_s = 30.0\ \text{N}$.

As T is increased further, a limiting value is reached at which the block breaks away and starts to move.

In diagram (V), a pull of 40.0 N acts on the rope. The maximum static frictional force that can act between the surfaces is $(f_s)_{max} = 40.0\ \text{N}$. The force to the right equals the force to the left placing the crate into a *state of impending motion*. The crate is on the verge of breaking free and moving to the right.

In diagram (VI), the tension in the rope is 50.0 N. The maximum force to the left is 40.0 N. $T + (f_s)_{max} = 50.0\ \text{N} - 40.0\ \text{N} = 10.0\ \text{N} = R$. A resultant of 10.0 N acts to the right since $R > 0$. The system is *NOT* in equilibrium. Note the sign of −40.0 N. Friction acts to the left.

Once in motion, it is found that the frictional force between the contact surfaces decreases. This new frictional force is proportional to the normal and to a proportionality factor called the *coefficient of kinetic or sliding friction*, μ_k. Maximum kinetic friction f_k is found by $\vec{f}_k = \mu_k \vec{N}$.

Frictional forces are parallel to the contact surfaces and directly oppose motion of the surfaces across one another.

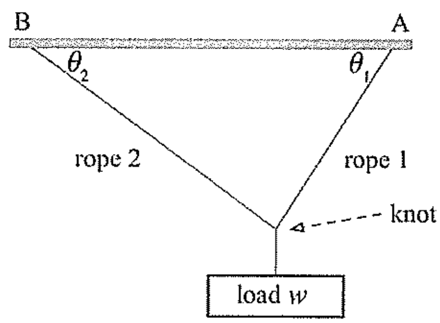
> **AP Tip**
>
> The force of static friction is greater than the force of sliding friction for the same materials. In symbols: $\vec{f}_s > \vec{f}_k$ and $\mu_s > \mu_k$.

The coefficient of sliding friction is independent of the velocity. Static and sliding friction are both nearly independent of the contact area.

Ropes Supporting a Load

(*College Physics* 9th ed. pages 100–101/10th ed. pages 102–103)

Consider the following system that is at rest. Since nothing is moving, the system is in a state of static equilibrium. Three ropes come together at the knot. Knots always make a great point to attach a frame of reference. The short rope supports the load of weight, *w*. Rope 1 and rope 2, attached to the ceiling at points A and B, support the entire system. How can we find the tension in the ropes if the load weighs 100 N?



Problem Solving Strategy

Make a simple sketch showing the system under consideration.

Identify each of the forces acting on the object. The two types of forces that usually act on bodies in equilibrium are *contact forces* and *field forces*. Both must be considered in a free-body diagram. The gravitational attraction exerted on the body by the Earth is *weight* and it does not have a point of contact with the body. Nonetheless, it exerts a real force on the body and must be considered an important factor. The direction of the weight vector, without exception, is always vertically downward toward the center of the Earth.

Next, it is very helpful to redraw the forces making a free-body diagram. In this diagram you reduce the object of interest to a single point. Attach a frame of reference to that point on the free-body diagram.

At the origin of the frame of reference, place each of the force vectors identified in step 2. Place each vector with its tail at the origin.

Resolve each force vector into its *x*- and *y*-components and sketch them in to your diagram. Label each force and its components.

Apply the first condition for equilibrium summing the components along the *x*-axis and the *y*-axis. Write the equations for the first condition for equilibrium, $\Sigma x = 0$ and $\Sigma y = 0$. It is recommended that at first the student constructs and uses a table somewhat as follows.

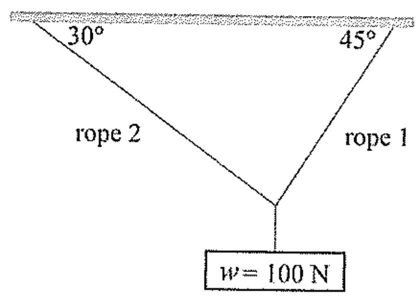
Finally, solve for the unknown forces.

At first this process appears lengthy, but it is necessary for a clear understanding of a problem.

Force	***x*-component**	***y*-component**
$F_1 @ \theta_1$	$F_1 \cos\theta_1$	$F_1 \sin\theta_1$
$F_2 @ \theta_2$	$F_2 \cos\theta_2$	$F_2 \sin\theta_2$
$F_3 @ \theta_3$	$F_3 \cos\theta_3$	$F_3 \sin\theta_3$
etc.	etc.	etc.
Sum of the components	$\Sigma x = x_1 + x_2 + x_3 + \ldots = 0$	$\Sigma y = y_1 + y_2 + y_3 + \ldots = 0$

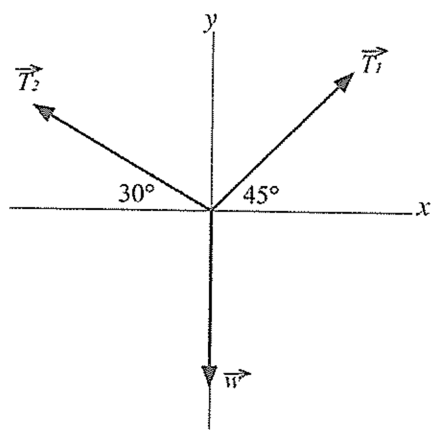
Sample Problem 6

In the system shown below, ropes 1 and 2 are attached to the ceiling at angles of 45° and 30° respectively. The system supports a 100 N load. Find the tensions in the ropes.

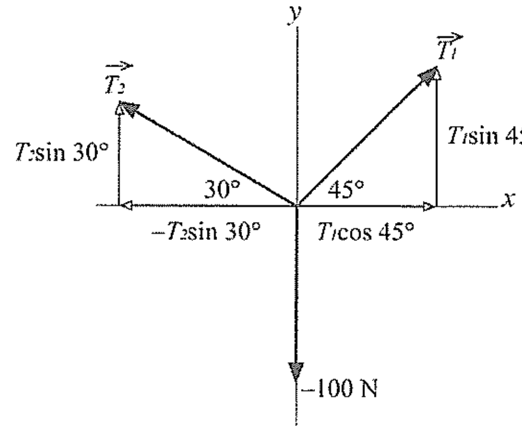


Solution to Problem 6

First sketch a free-body diagram making the knot the origin of the frame of reference.



Next, sketch in the x- and y-components of all the vectors and label them.



Force	***x*-component**	***y*-component**
T_1	$x_1 = T_1 \cos\theta_1$ $x_1 = T_1 \cos 45°$ $x_1 = 0.7071T_1$	$y_1 = T_1 \sin\theta_1$ $y_1 = T_1 \sin 45°$ $y_1 = 0.7071T_1$
T_2	$x_2 = T_2 \cos\theta_2$ $x_2 = T_2 \cos 150°$ $x_2 = -0.8860T_2$	$y_2 = T_2 \sin\theta_2$ $y_2 = T_2 \sin 150°$ $y_2 = 0.5000T_2$
w	0	–100 N
Component sum	$\Sigma x = x_1 + x_2 = 0$ $0.7071T_1 - 0.8860T_2 = 0$	$\Sigma y = y_1 + y_2 - 100 = 0$ $0.7071T_1 + 0.5000T_2 = 100$

We have developed a set of two simultaneous linear equations containing two unknowns.

(1) $0.7071T_1 - 0.8860T_2 = 0$

(2) $0.7071T_1 + 0.5000T_2 = 100$

Solving equation (1) for T_1 gives $0.7071T_1 = 0.8660T_2$. Substituting this into the second equation yields $0.8660T_2 + 0.5000T_2 = 100$ and $1.3660T_2 = 100$.

Then $T_2 = \mathbf{73.21\ N}$.

Writing equation (1) $0.7071T_1 - 0.8860T_2 = 0$, and replacing T_2 with $73.21\ \text{N}$, we now have $0.7071T_1 - 0.8660(73.21\ \text{N}) = 0$, and

$$T_1 = \frac{0.8660(73.21\ \text{N})}{0.7071} = \mathbf{89.67\ N}$$

There is a second approach to solving the problem that eliminates the need of a force table. In equilibrium we always use the first condition for equilibrium, $\Sigma x = 0$. We could also write that all of the forces to the right equal all of the forces to the left, or $\Sigma x_{right} = \Sigma x_{left}$.

From the above free-body diagram we can write $\Sigma x_{right} = \Sigma x_{left}$ and then $T_1 \cos 45° = T_2 \cos 150°$ which is $0.707T_1 = 0.866T_2$. Solving for T_1, $T_1 = \frac{0.866T_2}{0.707}$, we find that $T_1 = 1.225T_2$.

In equilibrium, $\Sigma y = 0$, or we could say that all the up forces must equal all the down forces, or $\Sigma y_{up} = \Sigma y_{down}$. Be careful, this is only true when a system is in equilibrium.

Returning to the free-body diagram, we write $\Sigma y_{up} = \Sigma y_{down}$ and adding $T_1 \sin 45° + T_2 \sin 150° = 100\text{ N}$.

We already have a value for T_1. Substituting into the first equation we have $T_1 \sin 45° + T_2 \sin 150° = 100\text{ N}$ and $(1.225T_2)\sin 45° + T_2 \sin 150° = 100\text{ N}$. Solving for T_2,

$$1.366T_2 = 100$$

$$T_2 = \frac{100}{1.366} = \mathbf{73.21\ N}$$

$$T_1 = 1.225T_2 = (1.225)(73.21\text{ N}) = \mathbf{89.67\ N}$$

TWO-BODY SYSTEM

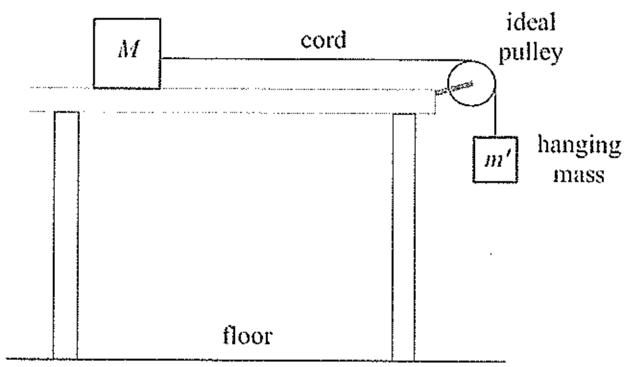
(*College Physics* 9th ed. pages 109–111/10th ed. pages 111–114)

SAMPLE PROBLEM 7

Consider a block of mass $M = 4.00$ kg at rest on a laboratory tabletop in a state of impending motion. A light cord of negligible mass is attached to M and runs over an ideal pulley to a hanging mass m'. The coefficient of friction for the contact surfaces is $\mu_s = 0.30$. What mass m' is required to place the system in its state of impending motion?

AP Tip

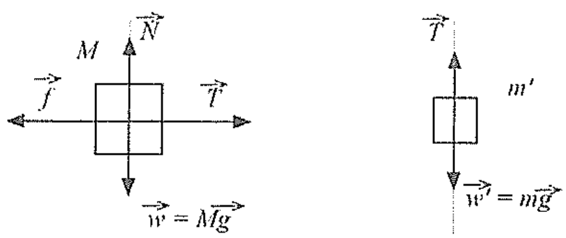
An *ideal pulley* is a one where we ignore any friction within the pulley itself. We also ignore its mass and its radius. An ideal pulley is a first approximation and our present goal is to investigate systems making first approximations. Later in the course, when considering rotational systems, the friction within the pulley, its mass and radius all become important factors.



SOLUTION TO PROBLEM 7

The system is in equilibrium and also in a state of impending motion. Just touching the cord or the masses will start the system in motion and it will no longer be at rest or in equilibrium. Make two free-body diagrams, one each for the masses M and m'.

Block M is in equilibrium vertically. The only motion it can have will be along the horizontal. We say that its motion is *constrained*, it can only move horizontally. All the up forces on M will equal all the down forces. The normal force, N, will equal the weight $w = Mg$.



$$N = w = Mg = (4.00\text{ kg})\left(9.8\ \text{m}/\text{s}^2\right) = 39.20\text{ N}$$

Horizontally, all of the forces to the right equal all the forces to the left. The frictional force between the contact surfaces will equal the tension in the rope, $T = f$. The magnitude of the tension T in the rope is everywhere the same. The ideal pulley simply changes the direction of the tension vector.

$$T = f = \mu N = (0.30)(39.20\text{ N}) = 11.76\text{ N}$$

The cord attached to the hanging mass supports the mass m'. The sum of all the upward forces acting on m' equals the sum of all the downward forces, or $T = w' = m'g$. Solving for m',

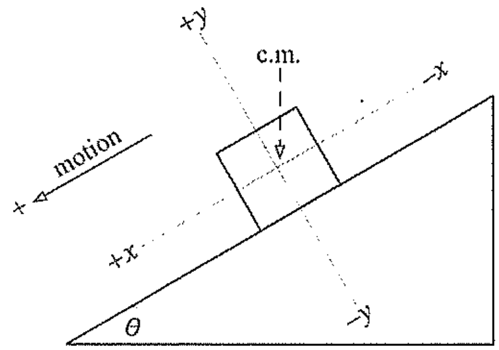
$$m' = \frac{T}{g} = \frac{(11.76\text{ N})}{\left(9.8\ \text{m}/\text{s}^2\right)} = \mathbf{1.20\ kg}$$

Only a 1.20 kg mass will maintain a state of equilibrium and a state of impending motion. A mass greater than 1.20 kg will increase the tension in the cord and will exceed the maximum frictional force between the block and the surface of the laboratory table.

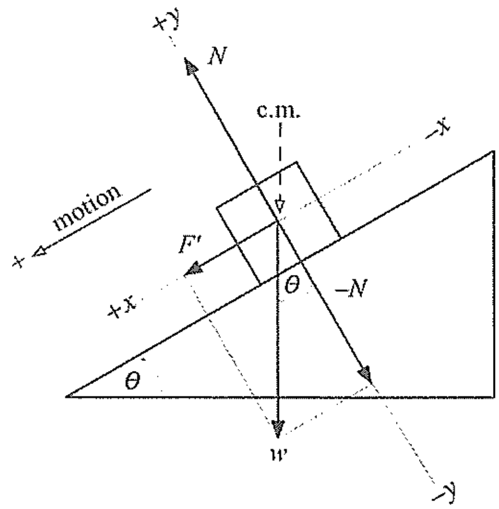
THE INCLINED PLANE

(*College Physics* 9th ed. pages 107–108/10th ed. pages 111–114)

The inclined plane is a simple machine that features a number of excellent learning ideas. Inclined planes are tilted at some angle θ.



In dealing with inclined planes, we attach our frame of reference to the c.m. of the body and make the x-axis parallel to the surface of the inclined plane and the y-axis perpendicular to that plane. We will also treat motion down the plane as being positive making the x-axis positive down the plane.



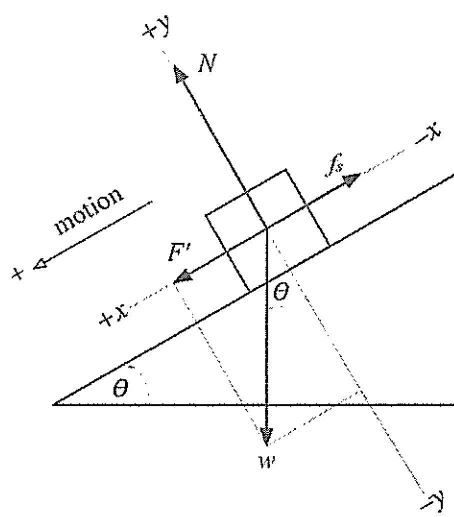
The weight of a body is a force w directed vertically downward. It is useful in analyzing the behavior of a body on an inclined plane to resolve w into vector components in directions parallel and normal to the inclined plane.

The gravitational force on the body is $w = mg$. Resolving w into its two components gives one component parallel to the plane and directed down the plane. This component, F', will be called the *effective weight,* and its value is $F' = mg\sin\theta$. Giving the inclined plane-body system personification, the object on the plane "thinks" its effective weight is its weight. *Effectively,* this is the force that *"pulls"* the body down the inclined plane.

The other component is normal to the plane and has the value $mg\cos\theta$. This component must be balanced by the plane's normal reaction force *N*. For this reason we could call the *y*-component, –*N*, the anti normal. The *normal force* that the inclined plane exerts on the body is $N = mg\cos\theta$. Again being guilty of personification, the inclined plane "thinks" that the anti normal force is the weight of the body.

Earlier, we define the force of friction as $f = \mu N$. On inclined planes the normal is $N = mg\cos\theta$ and we can say that the frictional force a body experienced on an inclined plane is then $f = \mu mg\cos\theta$.

Before we go much further we need to address something about the tilt angle or angle of inclination of an inclined plane. Somewhere between 10° and 20° there is a critical angle where for a given set of surfaces the effective weight of the body equals the maximum static frictional force. Below this angle, friction is greater than the effective weight. Above that angle the effective weight always exceeds friction and there will be motion down the plane. This angle is called the *angle of repose* or simply the *slip angle*. Keep in mind that the slip angle is unique for a given system.



At the slip angle, $F' = (f_s)_{max}$.

$$mg\sin\theta = \mu_s mg\cos\theta$$

Mass *m* and *g* are common to both sides of the equation and divide out leaving us with

$$\sin\theta = \mu_s\cos\theta$$

Solving for the coefficient of static friction gives

$$\mu_s = \frac{\sin\theta}{\cos\theta}$$

The sine of an angle divided by the cosine of the same angle lead to an identity called the tangent function or

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

If we know the slip angle, we can find the coefficient of static friction for the surfaces in the problem, or

$$\mu_s = \tan(\text{slip angle})$$

Or the slip angle is the inverse tangent of the coefficient of static friction, or

$$\text{slip angle} = \tan^{-1} \mu_s$$

SAMPLE PROBLEM 8

A 4.0 kg block sits at rest on an inclined plane. The coefficient of static friction for the surfaces is 0.268.

(a) Find the slip angle.

(b) What is the normal force the plane exerts on the block?

(c) What is the effective weight of the block?

SOLUTION TO PROBLEM 8

(a) The block is at rest making it in a state of static equilibrium. To find the slip angle we use $\text{slip angle}=\tan^{-1}\mu_s = \tan^{-1}(0.268) = \mathbf{15.0°}$. At all angles below 15.0° there can be no motion on the plane. At angles greater than 15°, friction cannot hold the block at rest and the effective weight "pulls" the block down the plane.

(b) The normal force is $N = mg\cos\theta = (4.0\text{ kg})\left(9.8\ ^{\text{m}}\!/_{\text{s}^2}\right)\cos 15.0° =$ **3.8 N**

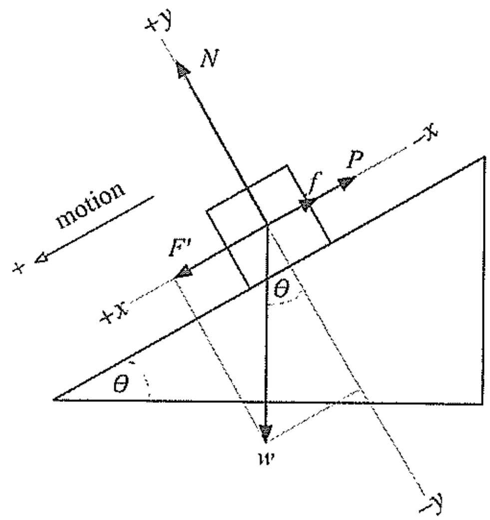
(c) The effective weight is $F' = mg\sin\theta = (4.0\text{ kg})\left(9.8\ ^{\text{m}}\!/_{\text{s}^2}\right)\sin 15.0° =$ **10.2 N**

In equilibrium on an inclined plane, motion is constrained to the plane. The body can stand still, slide down the plane at constant speed, or be pushed up the plane at constant speed. *All forces down the plane must equal all the forces up the plane.*

We will do one more thing, find the frictional force acting on the block. Remember, the block "wants" to slide down the plane making friction act up the plane. Friction on the plane is $f = \mu mg\cos\theta = (0.268)(4.0\text{ kg})\left(9.8\ ^{\text{m}}\!/_{\text{s}^2}\right)\cos 15.0° = 10.2\text{ N}$. Do all the forces up the plane equal the forces down the plane? Yes.

SAMPLE PROBLEM 9

A 10.0 kg box is lowered down a plane inclined at 30°. The coefficient of friction between the surfaces is 0.25. What push P up and parallel to the plane must be exerted on the box to lower it down the plane at constant speed?



SOLUTION TO PROBLEM 9

Look at the vector diagram given below. Lowering the box at constant speed places the box into a state of equilibrium and all forces acting down the plane must equal all the forces up the plane. Since the box slides down the plane, friction will act up the plane. The box cannot move along the y-axis, so we say its motion is constrained along the y-axis. The inclined plane prevents motion along the y-axis.

forces up the plane = forces down the plane

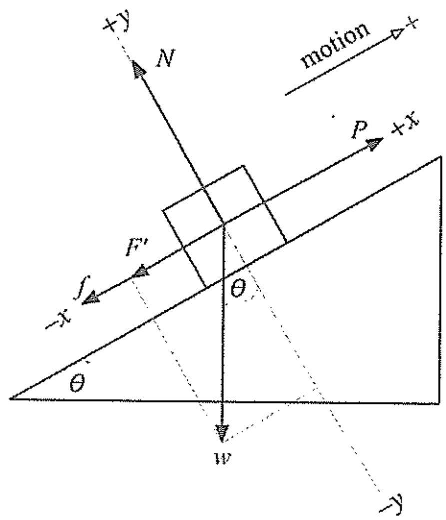
Therefore, $P + f = F'$ and

$$P = F' - f = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta)$$

$$P = mg(\sin\theta - \mu\cos\theta) = (10.0\text{ kg})\left(9.8\ \text{m}/\text{s}^2\right)(\sin 30° - 0.25\cos 30°) = \mathbf{27.8\ N}$$

Sample Problem 10

A 100.0 kg crate is pushed up a 35° incline at constant velocity. The coefficient of friction between the underside of the crate and the surface of the incline is 0.44. What push P parallel to the plane is required?



Solution to Problem 10

Constant velocity implies equilibrium and therefore forces up the plane = forces down the plane. Since motion exists up the plane, friction will act down the plane and we can write

$$P = F' + f = mg\sin\theta + \mu mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$$

$$P = mg(\sin\theta + \mu\cos\theta)$$
$$= (100.0\text{ kg})\left(9.8\ \text{m}/\text{s}^2\right)(\sin 35° + 0.44\cos 35°)$$
$$= \mathbf{915.3\ N}$$

Torque

(*College Physics* 9th ed. pages 235–239/10th ed. pages 240–244)

Up to this point we have seen that if all forces acting on a body intersect at a common point and their resultant is zero, the system must be in a state of translational equilibrium. But what happens if a body is acted on by *non-concurrent* forces? Now we have to consider the point of application of each force as well as the magnitude and direction of each force.

Consider a rod of length r that is attached to an external axle about which it can rotate freely. We call that point an *axis of rotation*. As shown below, a force F is applied to the opposite end of the rod. The distance from the axis of rotation to the point of application of the force is called the *moment arm*. Some refer to it as the *lever arm*. This is a "from and to" situation that makes the moment arm a vector, $\vec{r}$.

The applied force, $\vec{F}$, is non-concurrent with the axis of rotation. Non-concurrent forces tend to make bodies and systems rotate. A counterclockwise, CCW, rotation is considered to be a positive rotation (+) and a clockwise, CW, rotation is considered to be a negative rotation (–).

We define the torque $\vec{\tau}$, a vector, exerted by a force on a body as the measure of the effectiveness in turning a body about an axis of rotation. Torque is also called a *moment of force*. Mathematically, torque is defined as

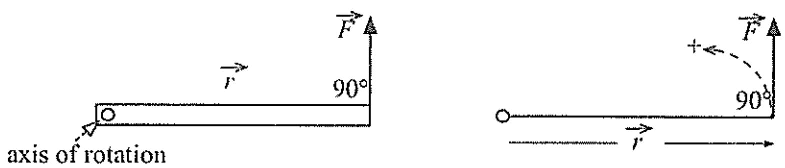
$$\tau = Fr\sin\theta$$

The angle θ is the angle between the moment arm and the applied force. When θ is 90°, when the moment arm and applied force are perpendicular, the $\sin 90° = 1$ and then

$$\tau = Fr$$

When θ is 0°, $\sin 0° = 0$ makes the line of action of the applied force concurrent with the axis of rotation and there is NO torque.

Note that the torque, a vector, is the product of two vectors $\vec{\tau} = \vec{F}\vec{r}$ and this vector equation is written as $\vec{\tau} = \vec{r}\vec{F}$ when working with deeper vector algebra. We will not be dealing with deeper vector algebra however.



SAMPLE PROBLEM 11

Considering the above drawings. A force of 100.0 N is applied at the end of the rod and at a 90° angle with the rod. Ignore the mass of the rod. If the rod has a length $r = 1.5\text{ m}$, what is the torque acting on the rod?

SOLUTION TO PROBLEM 11

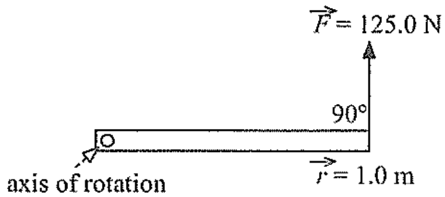
Note that the force is applied upward making the torque CCW(+). The torque about the axis of rotation is then

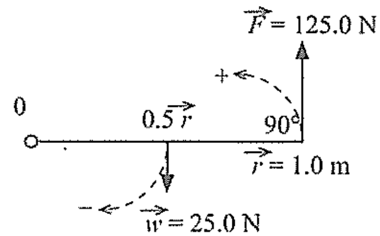
$$\tau_0 = Fr = +(100.0\text{ N})(1.5\text{ m}) = \mathbf{+150.0\ N \cdot m}$$

The system will rotate CCW.
The SI unit of torque is the $\text{N} \cdot \text{m}$.

SAMPLE PROBLEM 12

Consider the system illustrated below. A uniform rod, pivoted at one end, is 1.0 m long and weighs 25.0 N. Determine the torque about the axis of rotation.





SOLUTION TO PROBLEM 12

Since the rod is uniform, there is the weight of the rod to consider and its weight vector is at the center of mass or $0.5R = 0.5$ m. The weight vector tends to make the system rotate CW (–). The applied force on the system tends to make the system rotate CCW (+). The total torque involved is then

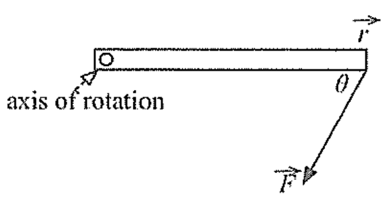
$$\Sigma\tau_0 = \tau_w + \tau_F = -w(0.5r) + Fr = -(25.0\text{ N})(0.5\text{ m}) + (125.0\text{ N})(1.0\text{ m})$$

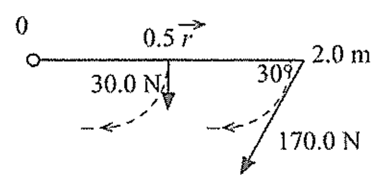
$$\Sigma\tau = +\mathbf{112.5\ N \cdot m}$$

The system will rotate CCW.

SAMPLE PROBLEM 13

A uniform rod, pivoted at one end, has a weight of 30.0 N and a length of 2.0 m. Find the total torque about the axis of rotation if the applied force at the end of the rod is 170.0 N





SOLUTION TO PROBLEM 13

Both the weight vector and the applied force tend to make the system rotate CW about the axis of rotation. The total torque is

$$\begin{aligned}\Sigma\tau_0 &= \tau_w + \tau_F \\ &= -w(0.5r) + (-F)r\sin 30^\circ \\ &= (-30.0\text{ N})(0.5 \times 2.0\text{ m}) + (-170\text{ N})(2.0\text{ m})(\sin 30^\circ)\end{aligned}$$

$$\Sigma\tau_0 = \mathbf{-200.0\ N \cdot m}$$

The negative sign implies that the system will rotate CW.

NON-CONCURRENT FORCES AND EQUILIBRIUM

(*College Physics* 9th ed. pages 240–247/10th ed. pages 245–252)

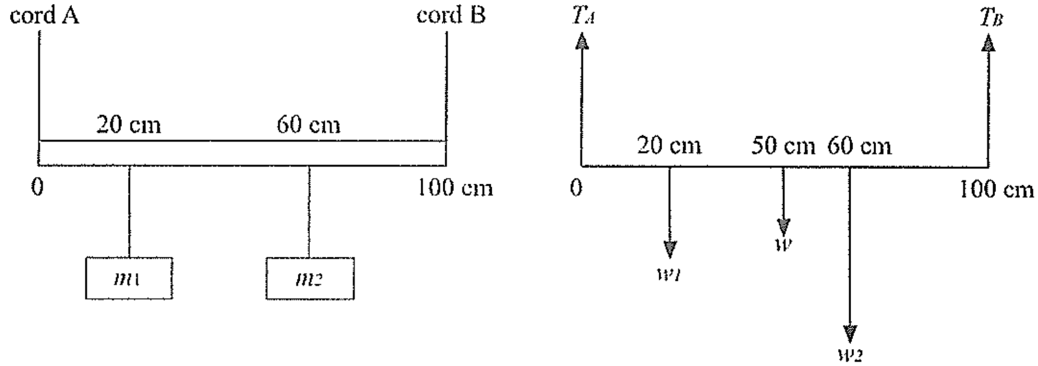
When there is no resultant torque acting in a system, $\Sigma\tau = 0$, the system is in a state of rotational equilibrium. We have already addressed the first condition for equilibrium where the sums of the x- and y-components add to zero, $\Sigma x = 0$ and $\Sigma y = 0$. We must now state the second condition for equilibrium and stipulate that there is NO resultant torque in the system or $\Sigma\tau = 0$.

SCAFFOLD MODEL OF ROTATIONAL EQUILIBRIUM

(*College Physics* 9th ed. pages 240–247/10th ed. pages 870–872)

SAMPLE PROBLEM 14

A uniform meterstick of mass $m = 0.10$ kg is supported at its ends by two cords as shown in the diagram below. A mass $m_1 = 0.80$ kg is suspended from the 20.0 cm mark and a mass $m_2 = 1.10$ kg is suspended at the 60.0 cm mark. The system is in equilibrium. Calculate the tensions in the cords.



SOLUTION TO PROBLEM 14

Beside the diagram of the system is a vector diagram of the forces involved. The forces are non-concurrent and the system is in a state of rotational equilibrium since non-concurrent forces tend to cause rotation. Both the first and second conditions for equilibrium hold. It is convenient for us to choose the zero, 0, end of the meterstick as our axis of rotation.

Study the vector diagram. There are no x-components in the system and the sum is $\Sigma x = 0$.

There are y-components. Weight is defined as $w = mg$. Each weight vector is negative since weight vectors always point downward toward the center of the Earth. The tension in each of the cords supporting the meterstick is directed upward and is considered positive.

There are two unknowns in the problem, T_A and T_B, and we can eliminate the twisting effect of one of them by making the left-end of the meterstick the axis of rotation. At the zero-end, our axis of rotation, T_A is concurrent and concurrent forces cannot produce a torque as there is no moment arm.

The system is in rotational equilibrium and $\Sigma y = 0$ or $\Sigma y_{up} = \Sigma y_{down}$.

Then $T_A + T_B = w_1 + w + w_2 = m_1 g + mg + m_2 g$

$$T_A + T_B = (0.80 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right) + (0.10 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right) + (1.10 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)$$
$$= 19.60 \text{ N}$$

Using the left-end or the 0 of the meterstick as the axis of rotation we sum all torques about the point, or $\Sigma\tau_0 = 0$. In rotational equilibrium we can also write $\Sigma\tau_{CCW} = \Sigma\tau_{CW}$. The tension vector T_B tends to make the system rotate CCW. The weight vectors all tend to make the system rotate CW.

We construct a force table as shown below and substitute in values.

Force	**x-component**	**y-component**	**Torque about O**
T_A	0	T_A	0
T_B	0	T_B	$+T_B r$ $+T_B(1.00 \text{ m})$ $+T_B$
w_1	0	$-m_1 g$ $(-0.80 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)$ -7.84 N	$(-m_1 g)(0.20 \text{ m})$ $(-0.80 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)(0.20 \text{ m})$ $-1.57 \text{ N}\cdot\text{m}$
w	0	$-mg$ $(-0.10 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)$ -0.98 N	$(-mg)(0.50 \text{ m})$ $(-0.10 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)(0.50 \text{ m})$ $-0.49 \text{ N}\cdot\text{m}$
w_2	0	$-m_2 g$ $(-1.10 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)$ -10.80 N	$(-m_2 g)(0.60 \text{ m})$ $(-1.10 \text{ kg})\left(9.8 \text{ m}/_{\text{s}^2}\right)(0.60 \text{ m})$ $-6.48 \text{ N}\cdot\text{m}$
Sum	$\Sigma x = 0$	$\Sigma y = T_A + T_B - 19.6 = 0$ $T_A + T_B = 19.60$ N	$\Sigma\tau_0 = +T_B - 8.54 = 0$ $T_B = 8.54$ N

Since the system is in rotational equilibrium and,

$\Sigma x = 0$, $\Sigma y = 0$ and $\Sigma\tau_0 = 0$

Adding the torques and solving for the unknown yields

$T_B =$ **8.54 N**

Adding the y-components and solving for the remaining unknown yields

$T_A = 19.60\text{ N} - T_B = 19.60\text{ N} - 8.54\text{ N} \approx \mathbf{11.06\text{ N}}$

We can do the same problem and eliminate the force table by solving by inspection.

CENTER OF MASS

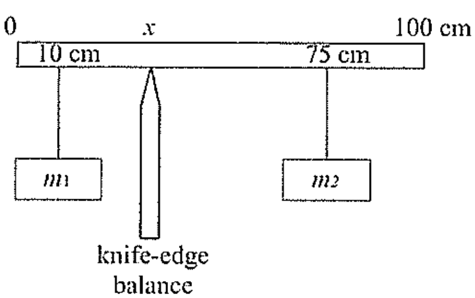
(*College Physics* 9th ed. pages 241–244/10th ed. pages 246–249)

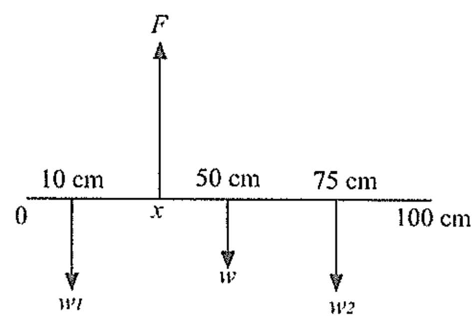
SAMPLE PROBLEM 15

A uniform meterstick has a weight $w = 4.0\text{ N}$. A weight of $w_1 = 6.0\text{ N}$ is attached to the meterstick at the 10 cm mark. Next a second weight $w_2 = 2.0\text{ N}$ is attached at the 75 cm point and then the entire system is balanced on a *knife edge*. Where is this balance point that is the center of mass of the system?

SOLUTION TO PROBLEM 15

First make a free-body diagram.





Since the system is balanced at its center of mass, it is in a state of equilibrium. The forces involved are non-concurrent and both conditions for equilibrium apply. There are no horizontal forces. We need only consider the up and down forces and the torques about the balance point.

Since all of the upward forces equal all of the downward forces when in equilibrium, the force F that the knife edge exerts on the system at the center of mass is the normal force N developed in the system and equals the total weight of the system, or

$$N = F = w_1 + w + w_2 = 6.0\text{ N} + 4.0\text{ N} + 2.0\text{ N} = 12.0\text{ N}$$

Treat the zero, 0, end of the meterstick as the axis of rotation for all of the torques in the system. All the CCW torques in the system equal all the CW torques or

$$Fx = w_1(10\text{ cm}) + w(50\text{ cm}) + w_2(75\text{ cm})$$

We will retain cm and express our answer in cm from the zero end.

$$(12.0\text{ N})x = (6.0\text{ N})(10\text{ cm}) + (4.0\text{ N})(50\text{ cm}) + (2.0\text{ N})(75\text{ cm})$$

$$x = \frac{410.0\text{ cm}}{12.0} = \mathbf{34.2\text{ cm}}$$

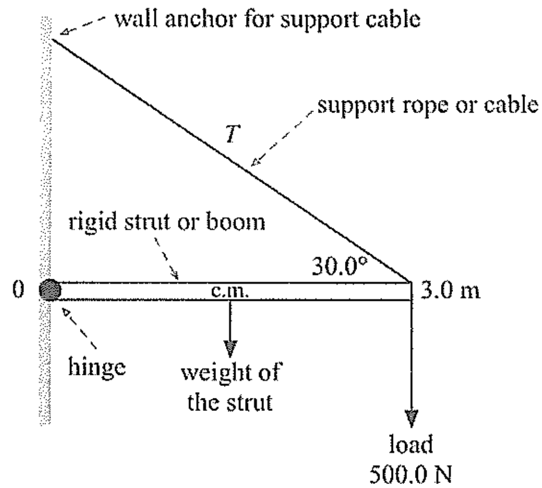
The center of mass of the system is at the 34.2 cm point.

THE BOOM CRANE

(*College Physics* 9th ed. pages 244–247/10th ed. pages 249–252)

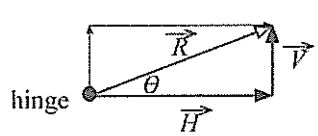
SAMPLE PROBLEM 16

The uniform *strut* shown in the *boom crane* illustrated below has a weight $w = 100.0\text{ N}$ and has a length of 3.0 m. A load of 500.0 N is attached to the end of the rigid strut. It is pivoted on the left at the hinge. A support cable of negligible weight is attached to the right end at a 30.0° angle with the horizontal. The crane is motionless. What reactionary force exists in the hinge?



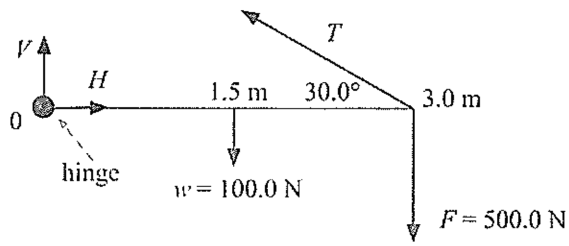
SOLUTION TO PROBLEM 16

Since the strut is uniform, its c.m. is at its center or 1.5 m left of the hinge. The reactionary force *R* has a horizontal component *H* and a vertical component *V*.



Note that the horizontal component *H* is the *thrust* generated by the hinge on the system. It is also the *compression* the system exerts on the strut. To find *R* we need to find *H* and *V*.

Next make a free-body diagram.



Since the system is motionless it is in a state of equilibrium. The forces are non-concurrent and both conditions for equilibrium will hold.

All the up forces equal all the down forces, or

$$V + T\sin 30.0° = w + F$$

$$V + 0.500T = 100.0\text{ N} + 500.0\text{ N}$$

$$V + 0.500T = 600.0$$

All the forces to the right equal all the forces to the left.

$$H = T\cos 30.0°$$

$$H = 0.866T$$

All of the CCW torques equal all the CW torques.

Treating the hinge as the axis of rotation eliminates the twisting effect of both H and V because they are concurrent and concurrent forces cannot produce torque.

$$T(\sin 30°)(3.0\text{ m}) = (100.0\text{ N})(1.5\text{ m}) + (500.0\text{ N})(3.0\text{ m})$$

$$(0.500)T(3.0\text{ m}) = 1650\text{ N}\cdot\text{m}$$

$$T = \mathbf{1100.0\ N}$$

From $V + 0.500T = 600.0$,

$$V = 600.0 - 0.500T = 600.0\text{ N} - 0.500(1100.0\text{ N}) = \mathbf{50.0\ N}$$

$$H = 0.866T = 0.866(1100.0\text{ N}) = \mathbf{952.6\ N}$$

With both H and V known,

$$R = \sqrt{H^2 + V^2} = \sqrt{(952.6\text{ N})^2 + (50.0\text{ N})^2} = 953.9\text{ N}$$

and

$$\theta = \tan^{-1}\left(\frac{V}{H}\right) = \tan^{-1}\left(\frac{50.0\text{ N}}{952.6\text{ N}}\right) = 3.0°$$

$$R = \mathbf{952.6\ N @ 3.0°}$$

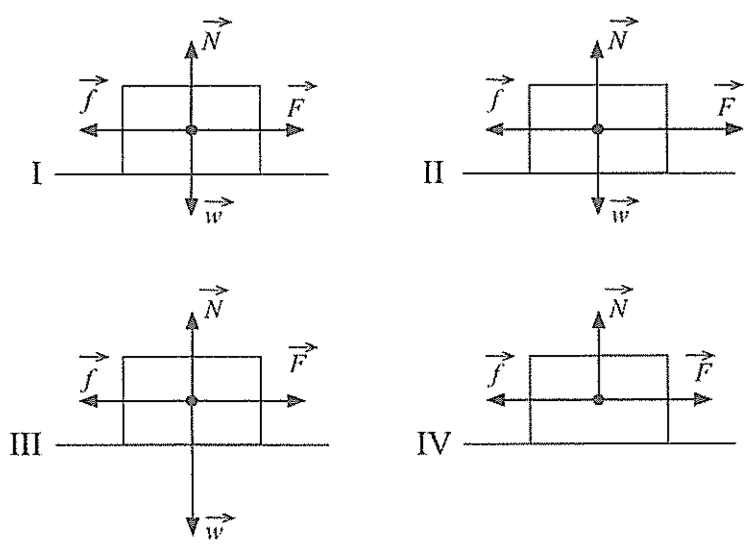
EQUILIBRIUM: STUDENT OBJECTIVES FOR THE AP EXAM

- ☐ You should be able to differentiate between mass and weight.
- ☐ You should be able to discuss gravitational intensity.

- You should be able to define equilibrium and give examples.
- You should be able to state and apply Newton's first and third laws of motion to equilibrium systems.
- You should be able to relate systems of concurrent forces and the first condition for equilibrium.
- You should be able to define equilibrant.
- You should be able to discuss the normal force.
- You should be able to differentiate between static and kinetic friction.
- You should be able to discuss contact forces and field forces.
- You should be able to define torque and give examples.
- You should be able to relate systems of non-concurrent forces and the First and Second Conditions for Equilibrium.
- You should be able to experimentally find the center of mass of a system.

MULTIPLE-CHOICE QUESTIONS

1. A 25.0 N force applied to a 100.0 N crate moves it at constant velocity on a flat horizontal surface where the coefficient of kinetic friction between the crate and the surface is 0.25.



   Which of the above illustrations correctly shows the forces acting on the body?
   (A) I
   (B) II
   (C) III
   (D) IV

2. A body cannot exert a force on itself. This statement is a direct consequence of Newton's
   (A) law of universal gravitation
   (B) first law of motion
   (C) second law of motion
   (D) third law of motion