15

WAVES AND SOUND

WAVES

(*College Physics* 9th ed. pages 455–456/10th ed. pages 464–465)

When a stone is dropped into a quiet pool of water, a disturbance is created where the stone enters the liquid. However the *disturbance* is not confined to the place of impact alone but spreads out so that it eventually reaches all parts of the pool. As the stone enters the water, it sets into motion the water molecules with which it comes into contact. These molecules set into motion neighboring molecules. They in turn produce similar motion in others, and so on, until the disturbance reaches water molecules at the edge of the pool. In this entire disturbance no molecules move far from their initial position. This is an example of a *mechanical* wave.

Only the disturbance moves through the water. This behavior is characteristic of all wave motions. The molecules move over short paths about their initial positions, and as a result a wave moves through the medium. *A mechanical wave is a disturbance that moves through a medium* in such a manner that at any point the displacement is a function of the time, while at any instant the displacement is a function of the position of the point. The medium as a whole does not progress in the direction of motion of the wave.

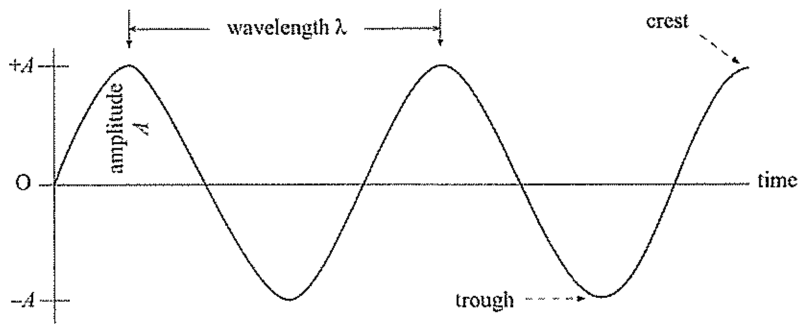
The motion of the wave through the medium is a result of the action of successive parts of the medium on each other. Such a wave can only travel in an *elastic medium*. If the molecules were entirely independent of each other, no waves could pass through.

There are several kinds of waves, their classification being made in accordance with the motion of the local part of the medium with respect to the direction of propagation. The most common types are *transverse* waves and *longitudinal* waves.

TRANSVERSE WAVES

(*College Physics* 9th ed. pages 456–463/10th ed. pages 465–472)

A sine wave is a repeating waveform that has amplitude, $\pm A$. The amplitude is the maximum displacement of the wave from equilibrium. The maximum positive displacement, $+A$, is called the crest of the wave. The negative displacement, $-A$, is the trough. The sine wave has a wavelength, λ. Wavelength is the distance between identical adjacent points on the wave. Distances are sometimes expressed in wavelengths.



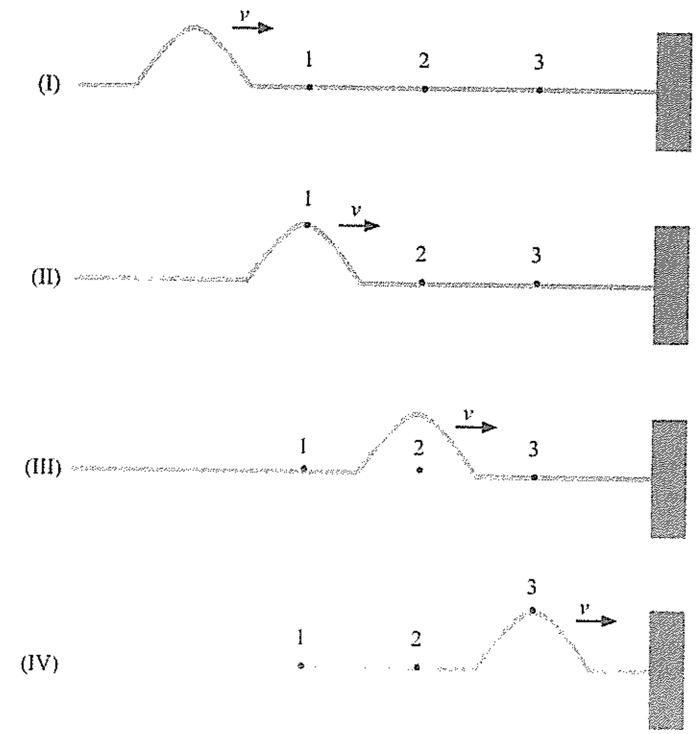
Sine Wave

In order for a mechanical wave to occur, it is necessary to have a *source* that produces a disturbance of some kind in an elastic medium through which the disturbance can be transmitted. An elastic medium behaves as if it were a succession of adjoining particles with each particle occupying an equilibrium position; if one of the particles is displaced, it is immediately subject to a restoring force as a result of interactions by neighboring particles, which in turn are subjected to reaction forces exerted by the original particle.

If one of the particles of the medium is given a sudden displacement by the source, this particle exerts forces on its immediate neighbors, which experience displacements; these immediately neighboring particles exert force on their neighbors, which also undergo displacements, and so on. In this way, the initial disturbance at the source causes a displacement wave to travel into the surrounding medium. Due to the inertia of the particles, the displacements of all particles do not take place at the same time; displacements far from the source occur later than the particles close to the source.

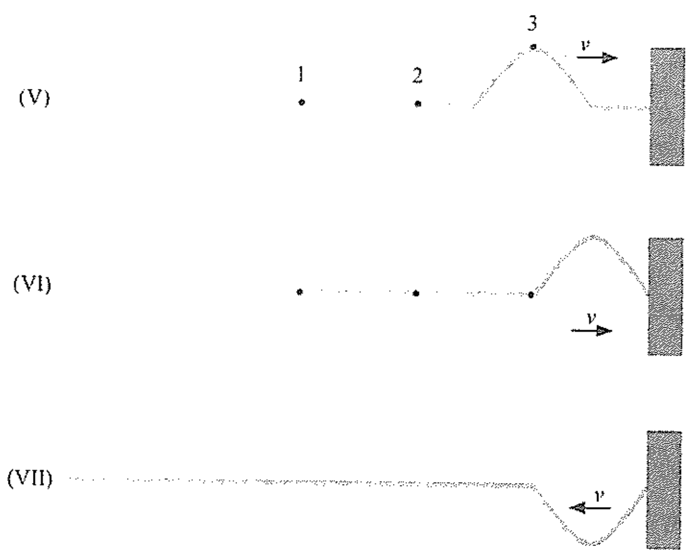
Consider a long, stretched string that is under tension, T, at the left. If the string is given a sudden upward displacement, a disturbance, the displaced portion of the string will exert forces tending to displace adjacent parts and at the same time will undergo forces tending to return the string to the undisturbed position. The result is a *pulse* that travels away from the disturbance.

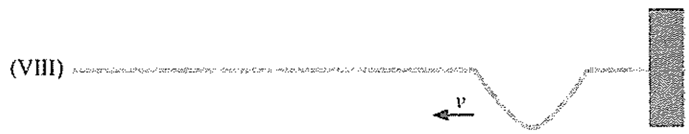
The pulse, as shown below, travels with some velocity, v, to the right. The velocity is a function of the medium and the tension the string is under. Note that in I, II, III, and IV, it is the pulse that moves along the string.

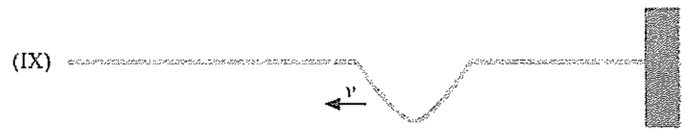


In VI and VII, the pulse collides with the barrier and is reversed in direction and the crest was inverted to become a trough. The wave travels with velocity $-v$ to the left. The passage of a crest or a trough along a stretched string is an example of a *transverse* wave motion.

Note: When a wave reaches a rigid and fixed termination it will experience a phase inversion. At a loose termination the wave will rebound in phase.





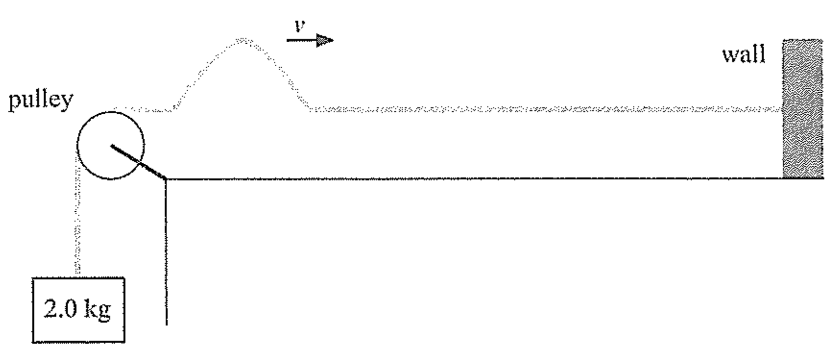


In a transverse wave the displacement of the particles of the medium are perpendicular to the direction of wave propagation. Each pulse will travel along the string until it reaches the end of the string. Let μ represent the mass of the string per unit length, or $\mu = \frac{m}{L}$. The tension in the string is T. We define the wave speed as

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}}$$

SAMPLE PROBLEM 1

A string 4.0 m in length has a mass of 3.0 g. The left end of the string is rigidly attached to a vertical wall. The other end hangs over a frictionless pulley with a 2.0 kg mass attached. What is the speed of a transverse wave in the string?



SOLUTION TO PROBLEM 1

The tension in the string is $T = mg = (2.0\text{ kg})\left(9.8\ {}^{\text{m}}\!/_{\text{s}^2}\right) = 19.6\text{ N}$

The wave speed $v = \sqrt{\frac{TL}{m}} = \sqrt{\frac{(19.6\text{ N})(4.0\text{ m})}{(0.0030\text{ kg})}} = \mathbf{160\ {}^{m}\!/_{s}}$

LONGITUDINAL WAVES

(*College Physics* 9th ed. pages 456–459/10th ed. pages 465–468)

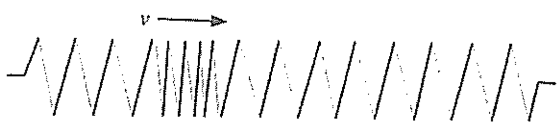
Another type of wave, which can occur in a helical coil, is shown below. Such a wave is called a *longitudinal wave*. In a longitudinal wave the displacements of the particles of the medium are parallel to the direction of propagation of the wave. Consider the stretched spring in diagram (a).

(a)

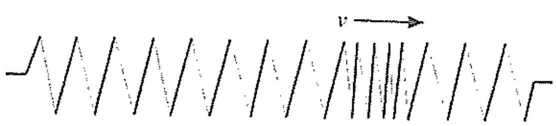
Suppose the left end of the spring is suddenly compressed by moving the end to the right. These coils exert forces on the adjacent coils, causing the compression to travel along the spring as a compression pulse. The speed of the wave is a function of the spring constant, k, and the mass per unit length, μ, of the spring. No part of the spring moves very far from its equilibrium position, but the longitudinal pulse continues to travel along the spring.

(b)

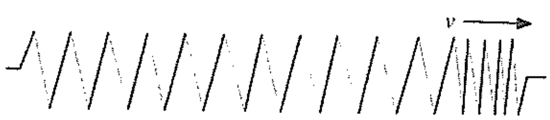
(c)



(d)



(e)



Note how the pulse travels along the spring to the right. The region where the coils are compressed is called a *condensation*. No part of the spring moves very far from its equilibrium position, but the pulse continues along the spring.

AP Tip

In a longitudinal wave the vibration of the individual particles is parallel to the direction of the wave propagation.

Had the coils on the left been forced apart, a *rarefaction* would have been formed. Upon removal of the force on the left, a longitudinal rarefaction pulse would have been propagated along the spring.

PROPERTIES OF WAVES

(*College Physics* 9th ed. pages 458–459/10th ed. pages 466–468)

Waves travel with a definite speed through a uniform medium. In wave motion, factors such as wave speed, wavelength, frequency, and amplitude must be considered. Some of these properties we have already discussed. Let us reiterate. The speed of a wave is the distance it advances per unit of time. The number of waves that pass a given point per unit time is called *frequency, f*, of the wave motion. The time it takes a single wave to pass a point is called the *period, T*, of the wave motion. The *wavelength,* λ, is the distance between two identical, adjacent points on the wave. The *amplitude, A*, of the wave is the maximum displacement from the equilibrium position.

When a wave passes from one medium into a second medium, the speed changes. In the process the frequency remains the same, but the wavelength changes in proportion to the wave speed; if v increases, λ also increases. This phenomenon is called *refraction*.

The relationship between wave speed, frequency, and wavelength are represented as

$$v = f\lambda$$

SAMPLE PROBLEM 2

A compression wave of frequency 250 Hz is set up in a steel rod and passes from the rod into the air. The speed of the wave in the steel rod is 4.88×10^3 m/s and 335.3 m/s in air. Determine the wavelength of the sound wave in both mediums.

SOLUTION TO PROBLEM 2

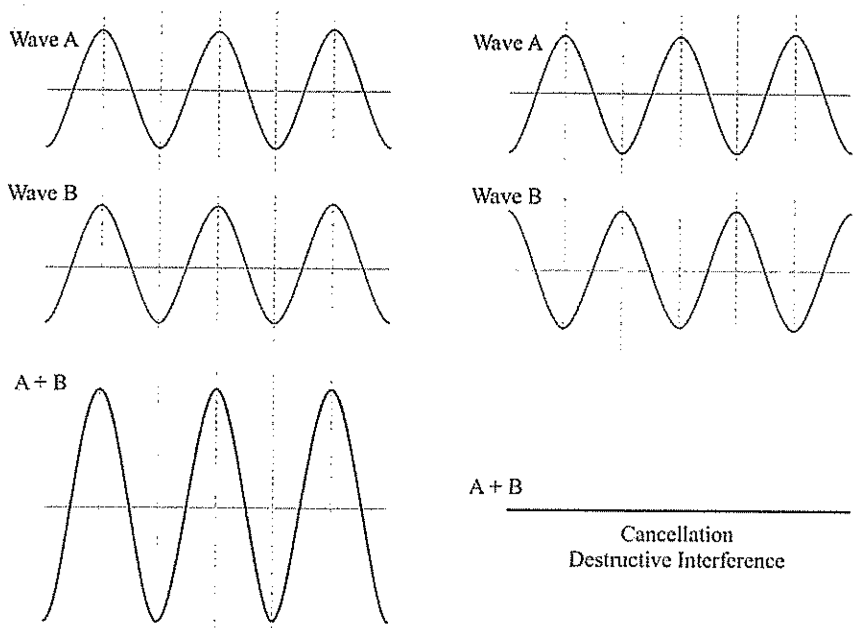
In steel the wavelength is $\lambda = \frac{v}{f} = \frac{4.88\times10^3\ \text{m/s}}{250\ \text{s}^{-1}} = \mathbf{19.5\ m}$

In air, the wavelength is $\lambda = \frac{v}{f} = \frac{335.3\ \text{m/s}}{250\ \text{s}^{-1}} = \mathbf{1.3\ m}$

SUPERPOSITION

(*College Physics* 9th ed. pages 461–463/10th ed. pages 470–472)

Suppose that two wave pulses, moments apart, are sent along a string that is under tension. The first wave, A, has been reflected and is on its way back and encounters the second oncoming wave, B. The two will interact. They are said to interfere. As they pass each other, their displacements will add and the displacement of the interfering waves is A + B. They simultaneously exist in the same medium, each wave traveling through the medium as though the other wave was not present. In the sense of propagation through the medium neither wave affects the other. However, at any point where two waves of the same kind reach simultaneously, the medium will have a displacement that is the *sum of the displacements of the individual waves*. If the two pulses are in phase, they will undergo *constructive interference*. When they are out of phase they suffer *destructive interference*.

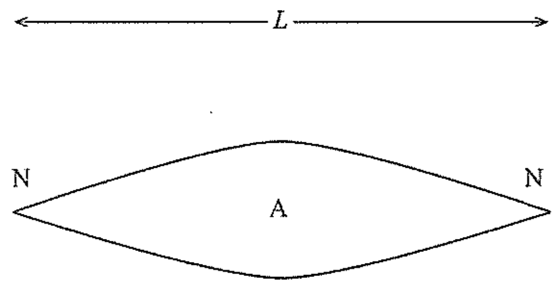


STANDING WAVES

(*College Physics* 9th ed. pages 489–493/10th ed. pages 498–503)

Consider the waves set up by a vibrating string whose end points are fixed, as in the following diagrams. The fixed end points represent *boundary conditions* that restrict the possible wavelengths that produce standing waves. Two identical waves moving in opposite directions superimpose to produce a large amplitude *standing wave*. It is called a standing wave because it represents a pattern or oscillation with time in a fixed location. Points on a standing wave where there is no displacement are called *nodes*, N. Halfway between two consecutive nodes are points where displacement is a maximum. We call these points *antinodes*, A.

Standing waves can occur at more than one frequency. The frequencies at which standing waves are produced are called *resonance* or *natural frequencies*. The lowest frequency at which a standing wave is produced is called the *fundamental frequency*. It is also called the *first harmonic*. Other frequencies are called *overtones*.



Fundamental mode or first harmonic.

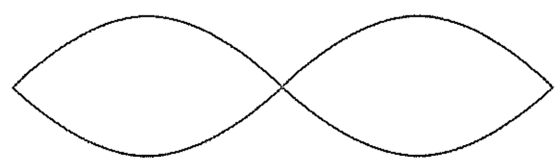
Higher modes of oscillation occur for shorter wavelengths. From the following diagrams it is noted that the allowable wavelengths are given by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, \cdots$$

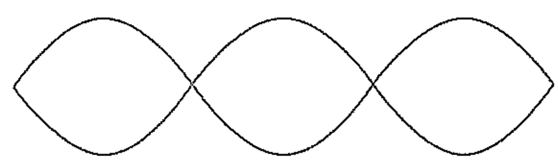
The lowest possible frequency is known as the *fundamental frequency*, f_1. The others are known as *overtones*. The frequency is determined from

$$f_n = n\frac{v}{2L} \qquad n = 1, 2, 3, \cdots$$

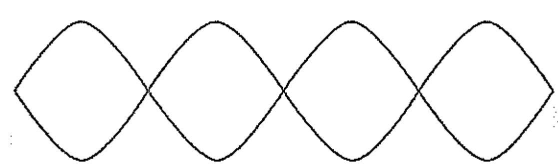
The entire series consisting of the fundamental and its overtones is known as a *harmonic series*.



First overtone or second harmonic.



Second overtone or third harmonic.



Third overtone or fourth harmonic.

Sample Problem 3

A 0.60 m long piece of steel piano wire has a mass of 10 g and is under a tension of 500 N. What is the frequency of its fundamental mode of vibration?

SOLUTION TO PROBLEM 3

The frequency of the fundamental standing wave is $f_n = n\frac{v}{2L}$. However we do not know the wave speed, v. Earlier we saw that the wave speed in a wire under tension is $v = \sqrt{\frac{TL}{m}}$. Combining equations gives

$$f_1 = \frac{n}{2L}\sqrt{\frac{TL}{m}} = \frac{1}{2(0.60\text{ m})}\sqrt{\frac{(500\text{ N})(0.60\text{ m})}{(0.010\text{ kg})}} = \mathbf{144\ Hz}$$

SOUND

(*College Physics* 9th ed. pages 473–478/10th ed. pages 481–485)

One of the most commonly observed types of mechanical waves is a sound wave. By means of a sound wave, tiny quantities of energy are carried to our ears and stimulate the nerves there to produce the sensation of sound. Usually the medium that transmits sound waves to our ears is the air that surrounds us. Air, as well as all gases, can transmit only longitudinal (compression) waves. As a sound travels it creates pressure and density variations in the air.

Although light and sound travel with a finite speed, the speed of light is so great in comparison that an instantaneous flash may be regarded as taking no time to travel many miles. When we see the light of a distant lightning flash we hear the sound of thunder later. The difference in time is due to the relatively low speed of sound.

As a general rule, sound travels faster in solids and liquids than it does in gases. Our air is a mixture of about 80% nitrogen and 20% oxygen. The speed of sound at sea-level, one atmosphere of pressure and 0°C is 341 m/s. The speed of sound increases as the temperature increases. The speed of sound can be approximated by

$$v = 331\,\text{m/s} + \left(0.6\frac{\text{m/s}}{\text{C}^\circ}\right)t$$

where t is the temperature measured in °C.

When a sound wave strikes a large, smooth, rigid surface, it will be reflected. An observer may receive two impulses, the wave directly from the source and the reflected wave. Humans have a *persistence* of hearing of about 0.1 s. If the direct wave and the reflected wave arrive more than a tenth of a second apart, they are perceived as two distinct sounds, and the reflected wave is called an *echo*. If the difference between the arrival times is less than a tenth of a second, the sounds merge into a blurred *reverberation*.

Sound travels approximately 34 m in a tenth of a second. Since the reflected wave must travel to the reflecting surface and back to the observer, and if the observer is 17 or more meters away from the reflecting surface, the echo will not be heard. Rooms in ordinary homes pose no echo or reverberation problem because their dimensions are on the order of three to five meters. Auditoriums and theaters, however, must be designed to control echoes and reverberations. A sound once started in an auditorium or a large room will persist by repeated reflection from the walls until the sound intensity is reduced to the point where it can no longer be heard. The repeated reflection of sound that results is called reverberation.

Sonar (sound navigation and ranging) uses echoes in water to determine depth and distances to nearby ships. Sonar equipment has become so highly refined that it is ultra-sensitive to small objects on the sea bottom.

VIBRATING AIR COLUMNS

(*College Physics* 9th ed. pages 495–498/10th ed. pages 504–508)

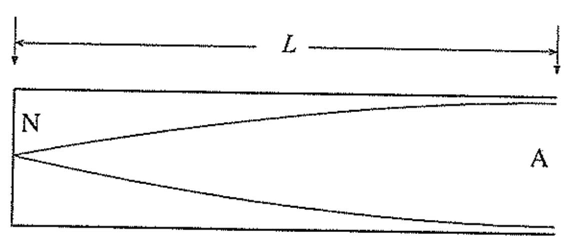
Earlier in the chapter we described the possible modes of vibration for a string bound at both ends. Sound can also be produced by the longitudinal vibrations of a column of air in an open or closed pipe. As in a vibrating string, the possible modes of vibration are determined by the boundary conditions. When a compression wave is set up in a closed pipe, the displacement of the air molecules at the closed end must be zero.

AP Tip

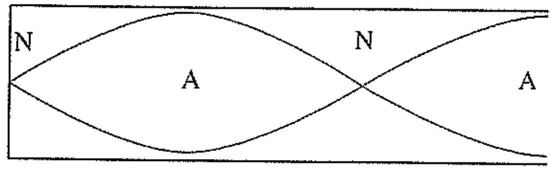
The closed end of a pipe must be a displacement node.

Standing waves in air are longitudinal in character and they are difficult to represent with any drawing or diagram. For convenience only, it is customary to indicate the positions of nodes and antinodes as if they were transverse standing waves.

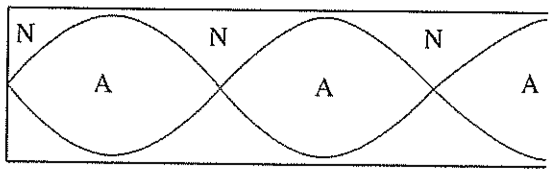
The various modes in which air columns may vibrate in open or closed pipes are shown in the following diagrams. With closed pipes, the lowest frequency is the fundamental, f_1, and the ones that follow are in odd multiples, $3f_1$, $5f_1$, $7f_1$, etc. No even-number harmonics can be sounded in a closed pipe. The wavelength of the standing wave will be $\lambda_n = \dfrac{4L}{n}$ and the frequency $f_n = \dfrac{nv}{4L}$.



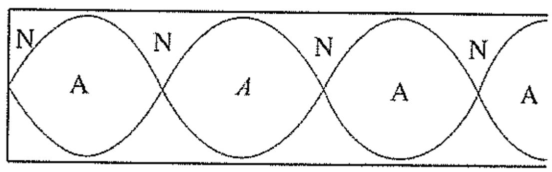
Fundamental mode or first harmonic.



First overtone or third harmonic.



Second overtone or fifth harmonic.



Third overtone or seventh harmonic.

Sample Problem 4

Determine the frequencies of the fundamental and the first three overtones for a 14 cm long closed pipe on a day when the speed of sound is 342 m/s.

Solution to Problem 4

First we need to find the fundamental and we do so by using $f_n = \frac{nv}{4L}$.

For the fundamental $n = 1$ and $f_n = \frac{nv}{4L} = \frac{1(342\ \text{m/s})}{4(0.14\ \text{m})} = \mathbf{611\ Hz}$.

The first, second and third overtones are the third, fifth and seventh harmonics.

First overtone $= nf_1 = 3(611\ \text{Hz}) = \mathbf{1833\ Hz}$

Second overtone $= nf_1 = 5(611\ \text{Hz}) = \mathbf{3055\ Hz}$

Third overtone $nf_1 = 7(611\ \text{Hz}) = \mathbf{4277\ Hz}$

AP Tip

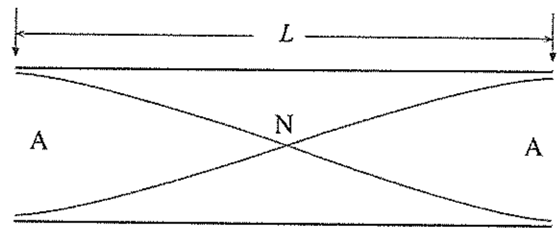
Only the odd harmonics are allowed for a closed pipe.

The air at the open end of a pipe has the greatest freedom of motion, and so the displacement is a maximum at the open end. The wavelength of the standing wave in the open pipe is $\lambda_n = \frac{2L}{n}$ and the frequency is $f_n = \frac{nv}{2L}$.

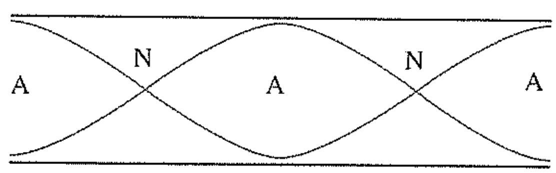
AP Tip

The open end of a pipe must be a displacement antinode.

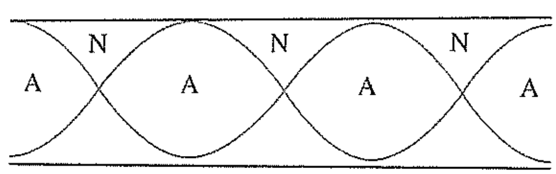
With an open pipe, the lowest possible vibration frequency is called the fundamental; the others, with whole-numbered multiples of the fundamental frequency, $2f_1$, $3f_1$, $4f_1$, etc., are possible harmonics.



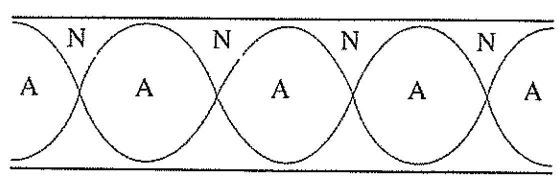
Fundamental mode or first harmonic.



First overtone or second harmonic.



Second overtone or third harmonic.



Third overtone or fourth harmonic.

SAMPLE PROBLEM 5

What is the speed of sound on a day when a 30 cm long open pipe has a frequency of 1200 Hz as its first overtone?

SOLUTION TO PROBLEM 5

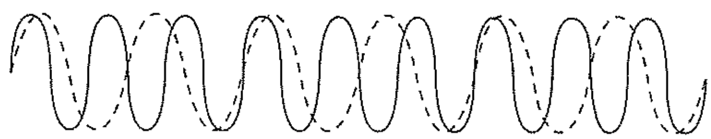
The first overtone in an open pipe is the second harmonic, so $n = 2$. For the second harmonic

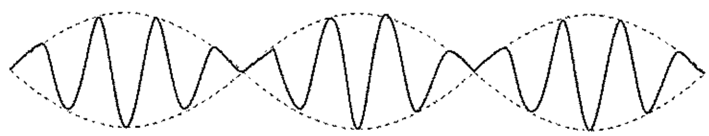
$f_n = \dfrac{nv}{2L}$ and solving for v, $v = Lf_2 = (0.30\text{ m})(1200\text{ Hz}) = \mathbf{360\text{ m/s}}$.

BEATS

(*College Physics* 9th ed. pages 499–500/10th ed. pages 508–509)

Consider two tuning forks whose frequencies differ only slightly in frequency. When they are struck simultaneously, they produce a sound that fluctuates in intensity, alternating between silence and a loud tone. The regular pulsations produced are called *beats*.





The superposition of the two sound waves produced by the vibrating tuning forks is the source of beats. The loud tones occur when the waves interfere constructively. The quiet tones are due to destructive interference. Observation shows that the number of beats, N, produced per second is given by the relationship $N = |f - f'|$. For example, tuning forks of frequencies 340 Hz and 343 Hz when struck simultaneously emit sound that pulsates 3 times per second.

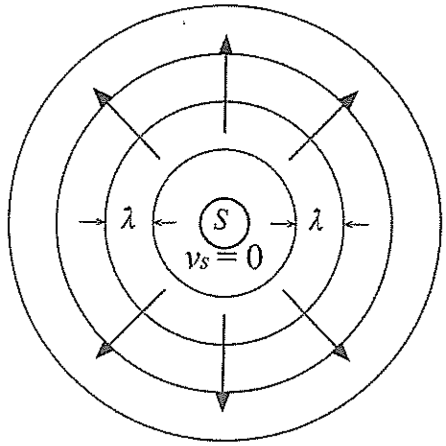
THE DOPPLER EFFECT

(*College Physics* 9th ed. pages 482–486/10th ed. pages 491–494)

When a sound source is moving relative to an observer, the pitch of the sound heard by the observer will not be the same as when the source is at rest. As the source approached the observer, the listener will hear a higher pitch than the one produced when the source is at rest. As the sound source recedes, the pitch is observed to be lower.

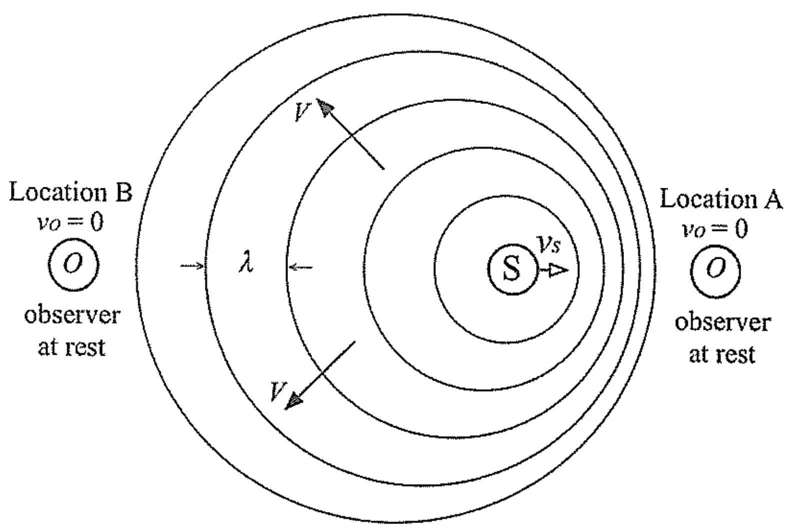
This phenomenon is not restricted to the motion of the source. If the source is stationary, an observer moving toward the source will hear a similar rise in pitch. A moving observer leaving the sound source will hear a lower-pitched sound. The change in frequency due to relative motion between a source and observer is called the *Doppler effect.*

The Doppler effect refers to the apparent change in frequency of a source when there is relative motion of the source and the observer.



Representation of sound waves being emitted from a stationary sound source S.

Consider the sound source moving to the right toward a stationary observer at location A. As the source emits sound waves, it tends to overtake the bunched up waves moving in the same direction. Each successive sound wave is emitted at a point that is closer to the observer than its predecessor. The result is that the wavelength, λ, keeps shrinking as the sound waves bunch. A smaller wavelength means a higher frequency. Observer A hears a higher pitch than observer B who hears a lower pitch.



The waves show a Doppler shift. The waves in front of a moving sound source are closer together than the waves behind the source.

V is the velocity of sound and f_S is the frequency of the sound source. The frequency the observer hears if f_O. The source moves with velocity v_S.

The velocity of sound in air is a function of the properties of air and is independent of the motion of the source. The frequency heard by a stationary observer from a moving sound source is given by

$$f_O = \frac{Vf_S}{V - v_S}$$

Where V is the velocity of sound, f_S is the frequency of the sound source, the frequency as heard by the observer is f_O and the speed of the sound source is v_S.

When the source is stationary, and the observer moves toward the source with speed v_O, the frequency heard by the observer is

$$f_O = \frac{f_S(V + v_O)}{V}$$

Sample Problem 6

A train whistle emits a sound with a frequency of 420 Hz. If the speed of sound is 336 m/s,

(a) what is the frequency of the sound heard by a stationary observer as the train moves toward that observer with a speed of 25 m/s?

(b) what frequency is heard as the train moves away from the observer at the same speed?

Solution to Problem 6

(a) The train approaches the observer making its speed positive. The frequency is given by

$$f_O = \frac{Vf_S}{V - v_S} = \frac{\left(336\ \mathrm{m/s}\right)\left(420\ \mathrm{s^{-1}}\right)}{\left(336\ \mathrm{m/s} - 25\ \mathrm{m/s}\right)} = \mathbf{454\ Hz}$$

(b) As the train recedes its velocity becomes negative making $v_S = -25\ \mathrm{m/s}$, and the frequency heard is found by

$$f_O = \frac{Vf_S}{V - v_S} = \frac{\left(336\ \mathrm{m/s}\right)\left(420\ \mathrm{s^{-1}}\right)}{\left[\left(340\ \mathrm{m/s}\right) - \left(-25\ \mathrm{m/s}\right)\right]} = \mathbf{387\ Hz}$$

If both the sound source and the observer are moving, the frequency heard by the observer becomes

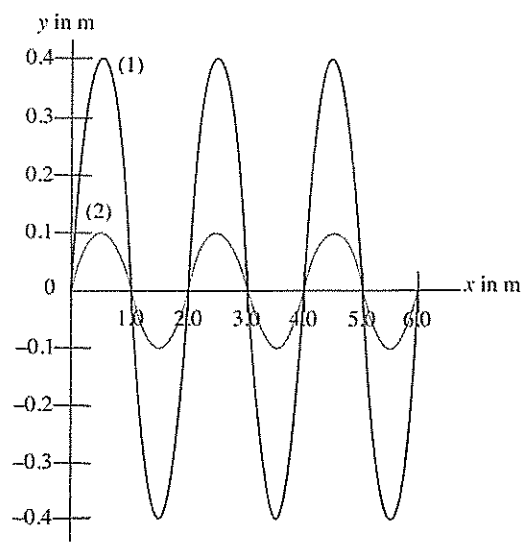
$$f_O = f_S \frac{V + v_O}{V - v_S}$$

WAVES AND SOUND: STUDENT OBJECTIVES FOR THE AP EXAM

- You should be able to explain why the velocity of transverse waves differs from the velocity of longitudinal waves in a given medium.
- You should be able to explain why longitudinal waves can propagate in a gas but transverse waves cannot.
- You should be able to explain why waves of different frequencies may have different velocities in a given medium.
- You should be able to explain why increasing the tension in a string increases the velocity of the transverse waves.
- You should be able to discuss how interference could result from the overlapping of two waves with somewhat different amplitudes.
- You should be able to explain the phenomenon of beat frequency.
- You should be able to explain why a cold organ pipe plays flat.
- You should be able to determine frequencies in open and closed pipes.
- You should be able to explain the Doppler effect.

MULTIPLE-CHOICE QUESTIONS

1. Two transverse waves (1) and (2) travel with the same speed.



Which of the following choices is a correct statement?
(A) Both waves (1) and (2) have equal amplitudes; however, the wavelength of (1) is four times larger than the wavelength of (2).
(B) The waves have the same amplitude, but the wavelength of (1) is 0.4 m and the wavelength of (2) is 0.1 m.
(C) Both (1) and (2) have equal wavelength; however, the frequency of (1) is four times greater than the frequency of (2).
(D) Both (1) and (2) have equal wavelengths; however, the amplitude of wave (2) is one-fourth that of wave (1).