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DC CIRCUITS

ELECTRIC CURRENT

(*College Physics* 9th ed. pages 590–596/10th ed. pages 600–606)

Certain materials, metals in particular, are better conductors of electricity than others. When a conductive path is provided between two points originally at different potentials, charge will move. Current is defined as the flow of charge q, and the intensity of the current, I, is defined as the time rate of flow of charge, or

$$I = \frac{q}{t}$$

The SI unit of current is the ampere, A. We define the ampere as $1\ \text{A} = 1\ \text{C/s}$.

The direction of current is a matter of convention. Since the concept of positive charge is basic to electrical terminology, the direction of conventional current is from positive (+) to negative (–). On the other hand, current in metallic conductors is a matter of electron flow, which is from negative (–) to positive (+).

AP Tip

The direction of conventional current is always the same as the direction that positive charges would move, even if the actual current consists of a flow of electrons.

When current through a conductor is always in the same direction, it is called direct current or DC.

AP Tip

Current means running or flowing and it is not proper to say that a "current flows." Current used in conjunction with the word *flows* is a redundancy.

Sample Problem 1

A charge of 22.0 C flows through a cross section of wire every minute.

(a) What is the current?

(b) How many electrons go through the cross section each minute?

Solution to Problem 1

(a) $I = \frac{q}{t} = \frac{22\ \text{C}}{60\ \text{s}} = 0.37\ \text{A}$

(b) $22.0\ \text{C} \times \frac{1\ e}{1.60 \times 10^{-19}\text{C}} = 1.38 \times 10^{20}\ e$

EMF

(*College Physics* 9th ed. pages 616–617/10th ed. pages 626–627)

Although electromotive force, (emf), $\mathcal{E}$, and potential difference, ΔV, are both measured in volts, there is a real distinction between them. The emf is defined as the work per unit charge done by a battery or a generator on the charges in moving them around the circuit. Potential difference between two points is defined as the work per unit charge done by the electrical forces in moving the charge from one point to the other. It is convenient to think of a source of emf as a kind of pump that acts on charge to bring it to a higher potential energy.

The SI unit of emf is the volt and it is defined as: $1\ V = 1\ \text{J/C}$. A seat of emf will perform 1 J of work on each 1-C charge that passes through it.

The most familiar source or seat of emf is the battery. A battery should be thought of as a *charge pump* that sends conventional current into a closed circuit. The seat of emf gives the current electrical energy and the resistors in the circuit use up that energy.

Resistance

(*College Physics* 9th ed. pages 596–600/10th ed. pages 606–611)

Resistance, R, is a physical characteristic of matter. Every material offers some resistance to electrical current. Good conductors like the metals copper, silver, and aluminum offer very little resistance to current. Nonconductive materials like rubber, plastic, and glass offer very high resistance to current.

The SI unit of resistance is the ohm (Ω) and is defined as $1\ \Omega = 1\ \text{volt/ampere} = 1\ \text{V/A}$.

OHM'S LAW

(*College Physics* 9th ed. pages 596–601/10th ed. pages 606–611)

Ohm's law is the fundamental law in electricity that makes it possible to determine the potential difference, V, across the ends of a resistor when the current, I, through it and its resistance, R, are known. Ohm's law for a resistor is

$$V = IR$$

AP Tip

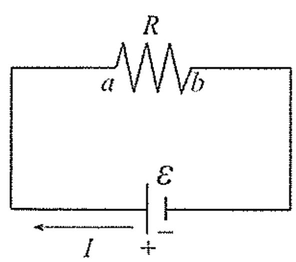
Circuit diagrams will greatly help in understanding a problem. Draw diagrams when working and solving problems.

SAMPLE PROBLEM 2

A battery is directly connected to a small light bulb. The battery maintains a potential difference of 6.0 V across the bulb. If the current in the circuit is 0.5 A, what is the resistance of the light bulb?

SOLUTION TO PROBLEM 2

First make a circuit diagram. The light bulb is a resistor and we show it in the circuit diagram as such.



The voltage drop across R is $V_{ab} = 6.0\ V$. So, $R = V / I = 6.0\ \text{V}/0.5\ \text{A} = 12\ \Omega$.

Although the resistance calculated in sample problem 2 is assumed to be the resistance of the light bulb, it really includes the resistance of the connecting wires and the resistance of the battery. In practice, we use wires of very low resistance, so this factor can be neglected in most calculations. Internal resistance in many sources of emf can be quite low and as a result can be neglected in some of our problem work.

We call the drop in voltage across the resistor an IR drop.

Power

(*College Physics* 9th ed. pages 601–603/10th ed. pages 611–614)

Electrical power is the rate at which energy is produced or used. Sources of emf do work on charge and give it energy. The power output of an emf source, measured in watts, is given by

$$P_0 = I\mathcal{E}$$

Resistors use electric energy, and the rate at which heat is dissipated in a circuit is called the power loss. The power loss in a resistor is given by

$$P = IV = I^2R = \frac{V^2}{R}$$

The power output of the seat of emf equals the power loss in the external circuit.

Sample Problem 3

A resistor uses 12.5 A when connected across 120 V.

(a) What is the resistance of this electrical component?

(b) Determine the power loss through the resistor.

Solution to Problem 3

(a) $R = V / I = 120\ \text{V}/12.5\ \text{A} = 9.6\ \Omega$

(b) $P = IV = (12.5\ \text{A})(120\ \text{V}) = 1.5\ \text{kW}$

Sample Problem 4

A 420-W resistor with a resistance of 60 Ω is used in a circuit.

(a) Determine the potential difference across the ends of the resistor.

(b) What current passes through the resistor?

Solution to Problem 4

Notice that the rules for the use of significant figures are not followed when using resistance in calculations since they are not calibrated to have significant figure values. Instead, they may have a tolerance of as much as ±20% so that the value of a 60-Ω resistor could range from between 48 Ω to 72 Ω. Be reasonable with your answers and do not exceed two decimal places in your problem work. Do any rounding off at the end of your calculations.

(a) $\text{P} = V^2 / R$ and $V = \sqrt{PR} = \sqrt{(420\ \text{W})(60\ \Omega)} = 159\ \text{V}$

(b) $I = V / R = 159\ \text{V}/60\ \Omega = 2.65\ \text{A}$

SAMPLE PROBLEM 5

A battery has an emf, $\mathcal{E} = 24.0\text{ V}$. When connected in a circuit it delivers a current of 3.4 A. The battery has an internal resistance of $1.5\ \Omega$.

(a) What is the terminal voltage?

(b) There is an external resistor in the circuit. What is the potential difference across this resistor?

(c) What is the power loss in this external resistor?

SOLUTION TO PROBLEM 5

(a) $V_T = \mathcal{E} - Ir = 24.0\text{ V} - (3.4\text{ A})(1.5\ \Omega) = 18.9\text{ V}$

(b) The external resistor must use whatever battery voltage the battery puts into the external circuit. The external resistor has a potential difference of 18.9 V.

(c) $P = IV = (3.4\text{ A})(18.9\text{ V}) = 64.3\text{ W}$

RESISTIVITY

(*College Physics* 9th ed. pages 597–599/10th ed. pages 607–609)

Several factors determine the resistance of any section of wire: (1) the length, L; (2) the cross-sectional area, A; and (3) the resistivity, ρ, a property of the material of which the wire is composed. The resistivity indirectly gives a measure of how well a current will be conducted through a piece of wire. Resistivity is related to resistance by the relationship

$$R = \rho \frac{L}{A}$$

SAMPLE PROBLEM 6

A 120-m long sector of circular wire has a diameter of 1.2 mm. The wire has a resistivity $\rho = 3.6 \times 10^{-8}\ \Omega \cdot \text{m}$.

(a) What is the resistance of this length of wire?

(b) What is the potential difference across the ends of the wire when a current of 2.4 A is sent through it?

SOLUTION TO PROBLEM 6

The radius of the wire is $r = 0.6 \times 10^{-3}\text{ m}$.

(a) Then

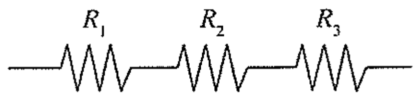
$$R = \rho \frac{L}{A} = \frac{(3.6 \times 10^{-8}\,\Omega \cdot \text{m})(120\text{ m})}{\pi (0.6 \times 10^{-3}\text{ m})^2} = 3.8\ \Omega$$

(b) $V = IR = (2.4\text{ A})(3.8\ \Omega) = 9.2\text{ V}$

RESISTORS IN SERIES

(*College Physics* 9th ed. pages 617–620/10th ed. pages 628–630)

When two or more resistances are connected in series in an electrical circuit, the current through all parts of the series combination is the same.



The sum of the voltage drops across the resistors in series is the sum of the voltage drop across each resistor. The equivalent resistance, R_{eq}, of the resistors in series is the sum of their resistances.

$$R_s = R_1 + R_2 + R_3 + \ldots$$

AP Tip

The current through resistors in series is the same in each resistor and the potential difference across them is additive.

SAMPLE PROBLEM 7

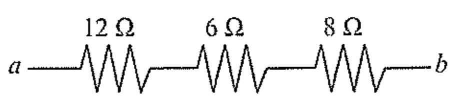
The three resistors, $R_1 = 12\ \Omega$, $R_2 = 6\ \Omega$ and $R_3 = 8\ \Omega$, are arranged in series.

(a) Find the equivalent resistance of the combination.

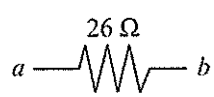
(b) The resistors are connected to a seat of emf. The current through the resistors is 2.4 A. What is the *IR*-drop across junctions *a* and *b*?

SOLUTION TO PROBLEM 7

(a) First, make a diagram.



For a set of resistors combined in a series, the equivalent resistance is the sum of the resistors as shown, and $R_s = R_{ab} = R_1 + R_2 + R_3 = 12\ \Omega + 6\ \Omega + 8\ \Omega = 26\ \Omega$. The three resistors behave like a single 26-Ω resistor.

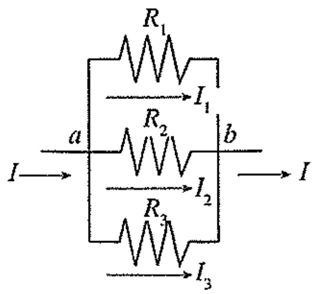


(b) The IR-drop between junctions *a* and *b* is $V_{ab} = IR_s = (2.4\ \text{A})(26\ \Omega) = 62.4\ \text{V}$.

RESISTORS IN PARALLEL

(*College Physics* 9th ed. pages 620–622/10th ed. pages 631–634)

When resistors are arranged in a parallel combination, the current splits. The sum of the currents in each branch is equal to the current that enters the combination. The voltage drop across the combination is equal to the voltage drop across each resistor in parallel.



The reciprocal of the equivalent resistance, R_{eq}, of the resistors in parallel is the sum of the reciprocals of the individual resistances.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

SAMPLE PROBLEM 8

The three resistors, $R_1 = 12\ \Omega$, $R_2 = 6\ \Omega$ and $R_3 = 8\ \Omega$, are arranged in parallel.

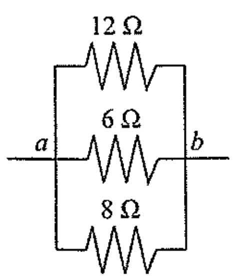
(a) Calculate their equivalent resistance.

(b) Junctions *a* and *b* are connected across an emf. If a current of 2.4 A enters junction *a*, what is the *IR* drop across junctions *a* and *b*?

(c) What is the current through each resistor in the parallel bank?

SOLUTION TO PROBLEM 8

(a) First, make a diagram of the resistor arrangement.



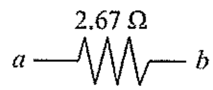
For a set of resistor combined in parallel, the reciprocal equivalent resistance is the sum of the reciprocals of the resistors.

$$\frac{1}{R_{eq}} = \frac{1}{R_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{8\ \Omega} = 0.375\ \frac{1}{\Omega}$$

Then

$$R_{eq} = R_{ab} = \frac{1}{0.375}\ \Omega = 2.67\ \Omega$$

The three resistors in parallel function as a single 2.67 Ω resistor as shown.



(b) Current $I = 2.4\ \text{A}$ enters the resistor at point a. The IR drop is
$V_{ab} = IR_s = (2.4\ \text{A})(2.67\ \Omega) = 6.4\ \text{V}$

(c) Each resistor in parallel undergoes the same voltage or IR drop, and that is 6.4 V. So, the current in R_1 is

$$I_1 = \frac{V_{ab}}{R_1} = \frac{6.4\ \text{V}}{12\ \Omega} = 0.53\ \text{A}$$

The current in R_2 is

$$I_2 = \frac{V_{ab}}{R_2} = \frac{6.4\ \text{V}}{6\ \Omega} = 1.07\ \text{A}$$

The current in R_3 is

$$I_3 = \frac{V_{ab}}{R_3} = \frac{6.4\ \text{V}}{8\ \Omega} = 0.80\ \text{A}$$

The sum of the currents going through the resistors in a parallel bank must equal the current entering the bank. As a check add the currents of the resistors in parallel

$$I = I_1 + I_2 + I_3 = 0.53\ \text{A} + 1.07\ \text{A} + 0.80\ \text{A} = 2.4\ \text{A}$$

They do check.

AP Tip

The largest resistor in parallel carries the smallest current and the smallest resistor in parallel carries the largest current. Current tends to take the path of least resistance.

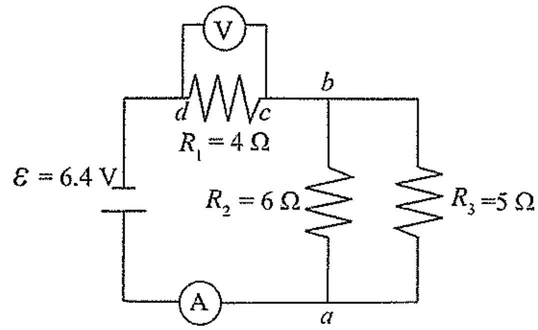
COMBINATION CIRCUITS

(*College Physics* 9th ed. pages 623–625/10th ed. pages 634–636)

In most electrical circuits, resistors are wired partly in series and partly in parallel. In such circuits, the rules for determining the equivalent resistance are applied to each part of the circuit for circuit analysis.

SAMPLE PROBLEM 9

The electrical circuit diagram below contains a seat of emf with zero internal resistance, three resistors, an ammeter, and a voltmeter. Ammeters are electrical instruments designed to measure electric current. An ammeter has low resistance and is connected in series so that all of the current passes through it. Voltmeters are designed to measure the potential difference across a resistor. Voltmeters have a high resistance and are connected in parallel across a resistor.



Perform the following analysis on the circuit.

(a) Find the total resistance of the circuit.

(b) What current will the ammeter A read?

(c) Determine the current in each resistor.

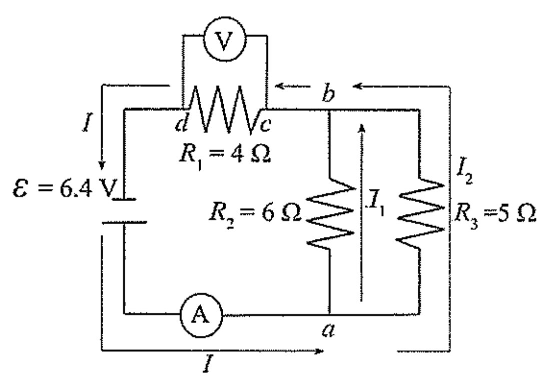
(d) What will the voltmeter V across resistor R_3 read?

(e) Calculate the power output of the battery.

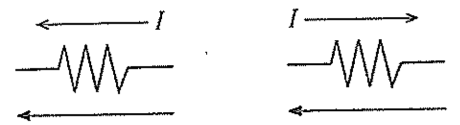
(f) Find the power drop in each resistor.

SOLUTION TO PROBLEM 9

Current I is issued from the positive terminal of the battery. At junction a, the current splits and current I_1 passes through R_2 and I_2 passes through R_3. I_1 and I_2 merge at junction b. The full current passes through the ammeter. See the figure below.



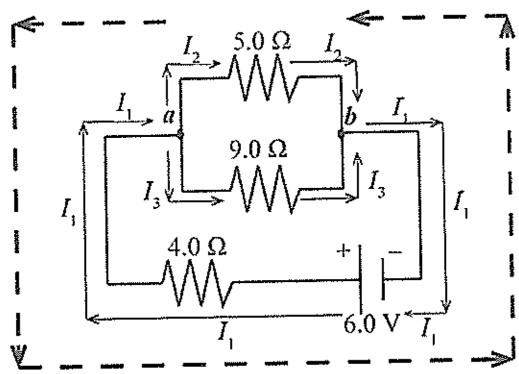
D. An IR-drop is considered negative (–) when the assumed current is against the direction of the trace.



Sample Problem 10

Consider the following electrical circuit. To two decimal places, determine currents I_1, I_2 and I_3 using Kirchhoff's rules. The battery issues conventional current I_1. The current passes through the 4.0 Ω resistor to junction **a**. At **a**, the current splits into currents I_2 and I_3. At junction **b**, the currents recombine back into I_1 that goes to the negative terminal of the battery. Whatever leaves the battery must return to the battery. We will start at junction **a** and trace around the circuit. Always keep in mind that we trace around the circuit counterclockwise, CCW. Be consistent at all times.

Trace starts at junction ***a*** and ends at junction ***b***.

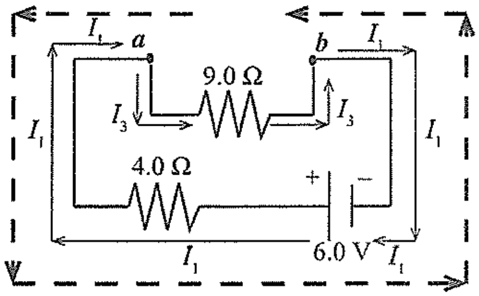


CCW trace from junction ***a*** to junction ***b***.

Solution to Problem 10

We trace CCW at junction **a** and note that the current runs contrary to the trace, making the signs of the currents negative. Since the current splits at junction **a**, we will apply the junction rule, and write $-I_1 = -I_2 - I_3$. Transposing the I_1 term gives us $I_1 - I_2 - I_3 = 0$. The junction rule is only used once.

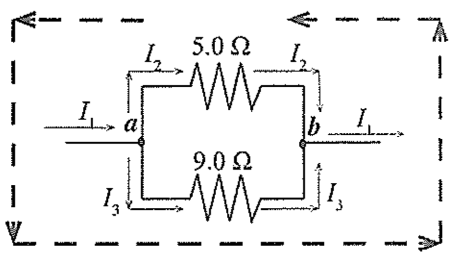
We will remove the upper resistor and our circuit becomes



CCW trace from junction *a* to junction *b*

Keep in mind that we trace CCW. With our first trace we are looking for a seat of emf. Tracing from junction **a** CCW we encounter the positive terminal of the battery making the *emf* negative, –6.0 V. Make a second trace looking for *IR*-drops across resistors. Going opposite to the trace the 4.0 Ω resistor has a negative *IR*-drop of $-4.0I_1$ and through the 9.0 Ω resistor the *IR*-drop is $-9.0I_3$. Since $\Sigma\mathcal{E} = \Sigma IR$ then we write $-6.0 = -4.0I_1 - 9.0I_3$.

Next, we take the loop containing the only 5.0 Ω resistor and the 9.0 Ω resistor.



CCW trace from junction *a* to junction *b*

Trace from junction **a** CCW around the loop looking for a battery, but there is none. The emf in the loop is zero. As we trace CCW from **a**, we are going with the current, making the *IR*-drop across the 9.0 Ω resistor $9.0I_3$. Continuing around the loop we encounter the 5.0 Ω resistor and note that we are tracing against the current making this *IR*-drop $-5.0I_2$. Equating the emf to the *IR*-drops yields $0 = -5.0I_2 + 9.0I_3$.

The equations from the junction rule and the loop rule has given us

$$I_1 - I_2 - I_3 = 0$$

$$-6.0 = -4.0I_1 - 9.0I_3$$

$$0 = -5.0I_2 + 9.0I_3$$

Placing the above set of simultaneous equations into better form gives

$$I_1 - I_2 - I_3 = 0$$

$$-4.0I_1 - 9.0I_3 = -6$$

$$-5.0I_2 + 9.0I_3 = 0$$

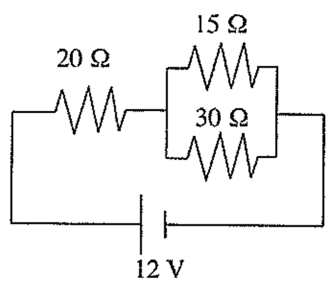
Solving the equations simultaneously yields, $I_1 = 0.83\text{ A}$, $I_2 = 0.55\text{ A}$ and $I_3 = 0.30\text{ A}$.

DC CIRCUITS: STUDENT OBJECTIVES FOR THE AP EXAM

- You should know the definition for electrical current in a conductor and be able to apply it.
- You should know that the direction of the current in a conductor is, by convention, the direction in which positive charge carriers flow.
- You should know and be able to apply Ohm's law and the power equations to all conductors as well as the entire DC circuit.
- You should know how to calculate the resistance of a conductor from its physical parameters.
- You should know how to draw and read schematic diagrams of the components in a direct-current circuit.
- You should know how to solve for equivalent resistance in series circuits, parallel circuits, and combination circuits.
- You should know how to solve for current, voltage, and power for an entire circuit and for each component of the circuit.
- You should know how to solve for terminal voltage in a DC circuit.
- You should be able to apply Kirchhoff's rules to more complex circuits.

MULTIPLE-CHOICE QUESTIONS

1. A simple circuit, shown in the diagram below, consists of a 20 Ω resistor in series with two resistors of 15 Ω and 30 Ω in parallel. The resistors are connected to a 12 V power supply. The current through the resistors is correctly identified in which of the following?



(A) $I_{15\,\Omega} = I_{20\,\Omega} = I_{30\,\Omega}$
(B) $I_{20\,\Omega} + I_{15\,\Omega} = I_{30\,\Omega}$
(C) $I_{20\,\Omega} = I_{15\,\Omega} + I_{30\,\Omega}$
(D) $I_{20\,\Omega} + I_{30\,\Omega} = I_{15\,\Omega}$

2. In reference to the above circuit in Problem 1, a voltmeter is placed across each of the resistors in turn and the electrical potential difference for each is indicated below. Which of the following gives the correct readings for the electrical potential difference across each of the resistors?

	20 Ω	15 Ω	30 Ω
(A)	6 V	6 V	6 V
(B)	8 V	8 V	4 V
(C)	8 V	4 V	8 V
(D)	8 V	4 V	4 V