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# ELECTROSTATICS

## ELECTRICAL CHARGE

(*College Physics* 9th ed. pages 513–515/10th ed. pages 523–525)

The basic components of all matter are the electron, the proton, and the neutron. The electron carries a negative charge, and the proton carries a positive charge. The neutron is electrically neutral. Charge is quantized which means that every electron in nature carries the same charge as all other electrons and every proton has the same charge as every other proton.

The SI unit of charge is the coulomb, C. The charge of the electron is $-1.60\times10^{-19}$ C, and the charge of the proton is $+1.60\times10^{-19}$ C.

### SAMPLE PROBLEM 1

A metal sphere is given a charge of 1.0 C. How many electrons were removed from the sphere?

### SOLUTION TO PROBLEM 1

The elemental charge is $e=1.60\times10^{-19}\,\mathrm{C}$.

$$1.0\ \mathrm{C}\times\frac{1\mathrm{e}}{1.60\times10^{-19}\,\mathrm{C}}$$

Coulomb and coulomb divide out giving us

$$1\ \mathrm{C}=6.25\times10^{18}\mathrm{e}$$

We call the magnitude of this charge the elemental charge, *e*. The electron carries the charge $-e$ and the proton, $+e$. These elemental charge are also denoted as $e^-$ and $e^+$.

The masses of the elementary particles are

$$m_e = 9.109 \times 10^{-31}\,kg$$

$$m_p = 1.673 \times 10^{-31}\,kg$$

$$m_n = 1.675 \times 10^{-31}\,kg$$

The law of conservation of electrical charge states that the total charge existing in the universe is constant. In a system, the total charge before and after changes remains the same.

In atoms, the proton population of the nucleus equals the electron population that surrounds the nucleus. Atoms that have lost or gained electrons are called ions. An atom that has lost an electron(s), a positive ion, is called a cation.

$$A \rightarrow e^- + A^+$$

An atom that gains an electron(s), a negative ion, is called an anion.

$$A + e^- \rightarrow A^-$$

The usual units of an electrical charge range from several micro coulombs, $\mu C$, to several nano coulombs, nC, with values as follows

$$1\ \mu C = 1 \times 10^{-6} C$$

$$1\ nC = 1 \times 10^{-9} C$$

Charges larger than several hundred micro coulombs are difficult to produce and maintain. Very large charges are quite dangerous.

## CONDUCTORS AND INSULATORS

(*College Physics* 9th ed. pages 515–516/10th ed. pages 525–526)

All metal atoms have a common property: they easily lose their valence electrons. Metals may be thought of as positive ions embedded in a sea of free electrons. The electrons belong to the entire metal and are mobile; they are free to move. We call metals conductors of electricity.

Non-metals have structures that are either atomic or molecular. There are no free electrons. These materials tend to prevent the flow of electricity. We call them insulators.

## ELECTROSTATIC ATTRACTION, ELECTROSTATIC REPULSION, AND ELECTROSTATIC FORCE

(*College Physics* 9th ed. pages 517–522/10th ed. pages 870–872)

Electrostatic means electricity at rest, and the words attraction and repulsion refer to the forces charged bodies exert upon one another at a distance. Like charges repel one another and unlike charges attract.

**AP Tip**

Attractions and repulsions are forces. Forces are vectors and have both magnitude and direction.

Surrounding every positive charge is an electric field. We assign the positive charge to be a source of the electric field. Surrounding a negative charge is an electrical field that we assign to be a field sink; the field flows into the sink, the negative charge.

In electrostatic repulsion, like fields interact is such a way as to repel. Two positive charges repel one another and two negative charges repel one another.

In electrostatic attraction, unlike fields interact as to produce electrostatic attraction.

## COULOMB'S LAW

(*College Physics* 9th ed. pages 517–522/10th ed. pages 527–532)

The magnitude of the electrostatic force $F$ between charges $q_1$ and $q_2$ that are separated by distance $R$ is given by Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{R^2}$$

where $\varepsilon_o$ is a constant called the permittivity of free space and its value is

$$\varepsilon_o = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$$

The constant $\varepsilon_o$ is usually written in terms of a new constant $k$ called the Coulomb constant. And we define it as $k = \frac{1}{4\pi\varepsilon_o}$. In SI units, we will express the constant as: $k = 9\times 10^9\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2$. Coulomb's Law now takes the form

$$F = k\frac{q_1 q_2}{R^2}$$

### SAMPLE PROBLEM 2

Two fixed point charges, $q_1$=+3 $\mu$C and $q_2$=–8 $\mu$C, exert an electrostatic force of attraction of 540 N on one another.

(a) What is their separation?

(b) How many excess electrons are on point charge $q_2$?

### SOLUTION TO PROBLEM 2

Opposite charges attract, making the force between them electrostatic attraction.

Write Coulomb's Law and then solve for $R$.

(a) We can write Coulomb's law and then solve for $R$:

$$F = k\frac{q_1 q_2}{R^2}$$

$$R = \sqrt{\frac{kq_1q_2}{F}} = \sqrt{\frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times10^{-6}\ \text{C})(8\times10^{-6}\ \text{C})}{540\ \text{N}}} = \mathbf{0.02\ m}$$

(b) Next we determine the number of excess electrons

$$8\times10^{-6}\ \text{C}\times\frac{6.24\times10^{18}\,e}{1\ \text{C}} = \mathbf{5\times10^{13}}\ e$$

## SUPERPOSITION

(*College Physics* 9th ed. pages 519–522/10th ed. pages 530–532)

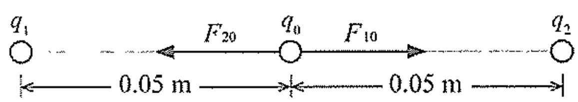
When a number of charges in a system act on a particular charge, each exerts an electrostatic force on that charge as well as on each other. These electrical forces are all calculated separately, one at a time, and are then added as vectors. This is called the superposition principle.

### SAMPLE PROBLEM 3

Two fixed point charges, $q_1 = +4$ nC and $q_2 = +6$ nC, are 10 cm apart. What is the force acting on a test charge, $q_0 = +1$ nC, place midway between $q_1$ and $q_2$?

### SOLUTION TO PROBLEM 3

Point charge $q_1$ exerts a force of electrostatic repulsion, $F_{10}$, to the right on the test charge $q_0$, and charge $q_2$ exerts electrostatic force, $F_{20}$, to the left of test charge $q_0$ as shown.




Use Coulomb's Law to determine $F_{10}$

$$F_{10} = k\frac{q_1 q_0}{R^2}$$
$$= \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\times10^{-9}\ \text{C})(1\times10^{-9}\ \text{C})}{(0.05\ \text{m})^2}$$
$$= 1.44\times10^{-5}\text{N at }0^\circ$$

We do the same for $F_{20}$

$$F_{20} = k\frac{q_1 q_o}{R^2}$$
$$= \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6\times10^{-9}\ \text{C})(1\times10^{-9}\ \text{C})}{(0.05\ \text{m})^2}$$
$$= 2.16\times10^{-5}\text{N at } 180°$$

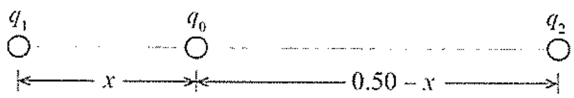
We can apply the superposition principle to find that these two forces are antiparallel, and their resultant is found by

$$F = F_{10} - F_{20} = 1.44\times10^{-5}\ \text{N} - 2.16\times10^{-5}\ \text{N} = -7.2\times10^{-6}\ \text{N}$$

The negative sign implies that the resultant force acts at 180°. The resultant force acting on $q_0$ is therefore $\mathbf{7.2\times10^{-6}}$ **N at 180°**.

## Sample Problem 4

Point charges $q_1 = -2$ nC and $q_2 = -5$ nC are 50.0 cm apart as shown. Where between the charges should a test charge $q_0$ be placed so that the resultant force acting on it is zero?




## Solution to Problem 4

The force $F_{10}$ acting on point charge $q_0$ by $q_1$ is found from Coulomb's law. The test charge will be placed a distance $x$ from $q_1$ and we write

$$F_{10} = k\frac{q_1 q_0}{x^2}$$

The force $F_{20}$ acting on point charge $q_0$ by $q_2$ is also found from Coulomb's law, and $q_0$ will be placed a distance $0.50-x$ from $q_2$. Thus

$$F_{20} = k\frac{q_2 q_0}{(0.5-x)^2}$$

If $F_{10} = F_{20}$, then

$$k\frac{q_1 q_0}{x^2} = k\frac{q_2 q_0}{(0.5-x)^2}$$

The terms $k$ and $q_0$ are common to both sides of the equation and divide out. After cross-multiplying, we have

$$q_1(0.5-x)^2 = q_2 x^2$$

Dividing both sides by $q_1$ and expanding gives

$$0.25 - x + x^2 = \frac{q_2}{q_1} x^2$$

$$0.25 - x + x^2 = \frac{\left(5\times10^{-9}\right)}{\left(2\times10^{-9}\right)} x^2$$

$$0.25 - x + x^2 = 2.5x^2$$

Transposing and simplifying yields $1.5x^2 + x - 0.25 = 0$. Notice that this is a quadratic equation of the form $ax^2 + bx + c = 0$ with solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1.5)(-0.25)}}{2(1.5)}$$

$$x = 0.194 \text{ m and } -0.860 \text{ m}$$

We wanted the position of $q_0$ between the two point charges; $q_0$ should be placed 0.194 m to the right of $q_0$. Notice that we did not need to know the value of $q_0$.

### Alternate Solution to Problem 4

An alternate solution uses $F_{10} = F_{20}$ and

$$k\frac{q_1 q_0}{x^2} = k\frac{q_2 q_0}{(0.5 - x)^2}$$

Here, $k$ and $q_0$ are common to both sides, so they divide out leaving

$$\frac{q_1}{x^2} = \frac{q_2}{(0.5 - x)^2}$$

Substituting in for the charge on both sides of the equation gives

$$\frac{2\text{ nC}}{x^2} = \frac{5\text{ nC}}{(0.5 - x)^2}$$

The nC units are common to both sides and divide out. So

$$\frac{2}{x^2} = \frac{5}{(0.5 - x)^2}$$

Taking the square root of both sides yields $\sqrt{2}/x = \sqrt{5}/(0.5 - x)$. Cross multiplying gives $1.414(0.5 - x) = 2.2311$, or $x = \mathbf{0.194\ m}$.

## The Electric Field

(*College Physics* 9th ed. pages 522–526/10th ed. pages 532–536)

Fields are modifications of space. Masses generate a gravitational field, or g-field. You cannot see or feel or smell the g-field of the Earth, but it is there. We can test to see if a g-field is present by taking a test body of mass $m_0$ and dropping it. The g-field of the mass interacts with the g-field of the Earth and it falls vertically downward under gravitational force. The gravitational force is the weight of the body, $F = w = mg$.

The electric field, $E$, is a vector with magnitude and direction. Surrounding all electrical charge are E-fields of intensity $E$. To test for an E-field, we place a positive test charge $q_0$ in the field and measure the electrostatic force $F$ acting on it. If $q_0$ is repelled, the charge creating the field is positive, and if $q_0$ is attracted, the charge creating the field is negative. We define the E-field as

$$E = \frac{F}{q_0}$$

The magnitude of the E-field at a point in space set up by a point charge can be calculated by using a modification of Coulomb's law where R represents the distance from the charge setting up the field and the point in question

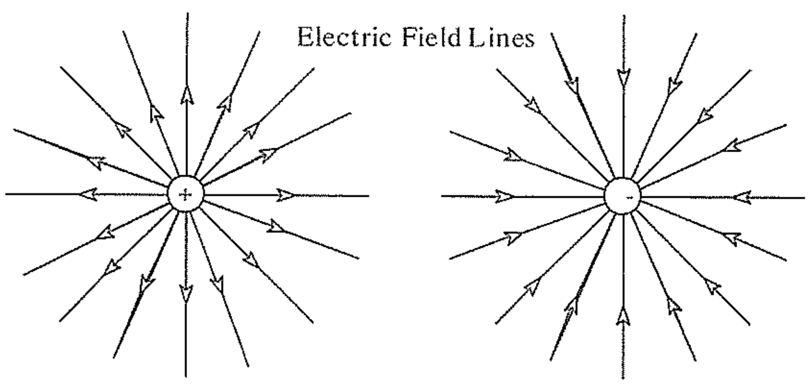
$$E = k\frac{q}{R^2}$$

The SI unit of the electric field is the newton per coulomb, N/C.

## LINES OF FORCE

(*College Physics* 9th ed. pages 526–528/10th ed. pages 536–538)

To aid in visualizing the electric field, the concept of lines of force is frequently used. A line at every point indicates, by its direction, the direction a unit charge would take if placed there, directly away from a positive charge and directly toward a negative charge.



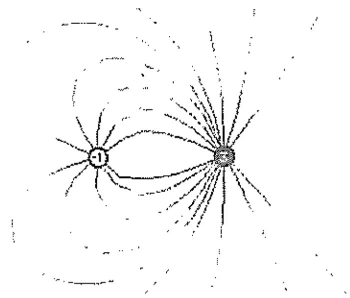

Positive charges behave as a source of an electrical field.

Negative charges behave as a sink for an electrical field.

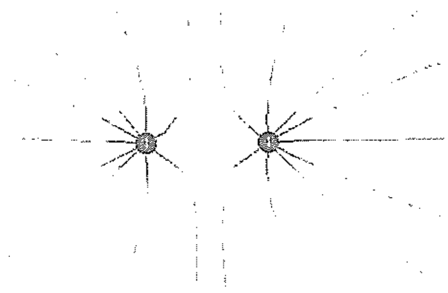
The direction of a line of force at any point in an electrical field is the same as the direction of the resultant E-field vector at that point. There are three rules to be followed when sketching electric field lines:

1. Lines of force never cross one another.
2. Lines of force in a grouping of point charges begin on a positive charge and end on a negative charge.

3. The number of lines leaving a positive charge or ending on a negative charge is proportional to the magnitude of the charges.



The lines of force associated with point charges $+3q$ and $-q$ illustrating electrostatic attraction.



The line of force associated with point charges $+q$ and $+q$ illustrating electrostatic repulsion.

## SAMPLE PROBLEM 5

A charge $q = +2\text{ nC}$ is placed in a uniform electrical field. In the field, $q$ experiences a force $F = 4\times10^{-4}\text{ N}$. What is the magnitude of the electrical field intensity?

## SOLUTION TO PROBLEM 5

Using the definition of the E-field we write

$$E = \frac{F}{q} = \frac{4\times10^{-4}\text{ N}}{2\times10^{-9}\text{ C}} = \mathbf{2\times10^{5}}\ \text{N/C}$$

## SAMPLE PROBLEM 6

Determine the magnitude of the electrical field 2.0 cm from a point charge $q = +10\text{ nC}$.

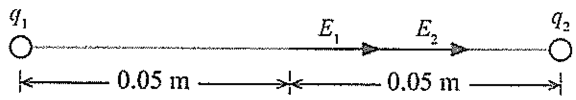
## SOLUTION TO PROBLEM 6

This time, we use the modification of Coulomb's law and substitute

$$E = k\frac{q}{R^2} = \frac{\left(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10\times10^{-9}\ \text{C}\right)}{\left(0.02\ \text{m}\right)^2} = \mathbf{2.25\times10^{5}}\ \text{N/C}$$

## SAMPLE PROBLEM 7

Two fixed point charges, $q_1 = +8\ \mu\text{C}$ and $q_2 = -2\ \mu\text{C}$, are 10 cm apart as shown. What is the electrical field intensity at the midpoint of the line connecting $q_1$ and $q_2$?




## SOLUTION TO PROBLEM 7

First, determine the electric field contributions due to each point charge.

$$E_1 = k\frac{q_1}{R^2} = \frac{\left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8\times10^{-6}\ \text{C}\right)}{(0.05\ \text{m})^2} = 29\times10^6\ \frac{\text{N}}{\text{C}}\ \text{to the right}$$

$$E_2 = k\frac{q_2}{R^2} = \frac{\left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2\times10^{-6}\ \text{C}\right)}{(0.05\ \text{m})^2} = 7.2\times10^6\ \frac{\text{N}}{\text{C}}\ \text{to the right}$$

Note: The directions of $E_1$ and $E_2$ are established by the charges on $q_1$ and $q_2$ and therefore the negative sign for $q_2$ need not be included in the calculation of $E_2$.

Applying the superposition principle gives

$$E = E_1 + E_2 = 29\times10^6\ \text{N/C} + 7.2\times10^6\ \text{N/C} = \mathbf{3.6\times10^7\ N/C\ to\ the\ right}$$

Field strengths, or field intensities, are important for a major reason: they allow us to find force. From $E = F/q$ it is easy to see that

$$F = Eq$$

## SAMPLE PROBLEM 8

Two charged, parallel metal plates have a gap of 4.0 mm and the E-field between the plates is 6000 N/C directed downward. An electron is released from rest from the negative plate.

(a) What force does the electron experience?

(b) Calculate the acceleration the electron experiences in the E-field.

## SOLUTION TO PROBLEM 8

(a) Since the plates cannot be considered as point charges in this problem, we cannot use the modification of Coulomb's law. We use the above equation instead and substitute:

$$F = Eq = Ee = \left(6000\ \text{N/C}\right)\left(1.60\times10^{-19}\ \text{C}\right) = \mathbf{9.6\times10^{-16}\ N}$$

Because the field is directed downward, the upper plate is positive and the force acting on the electron is upward. The force acting on the electron is therefore $\mathbf{9.6\times10^{-16}\ N\ at\ 90^\circ}$.

(b) The acceleration is found from Newton's second law:

$$F = ma \text{ and } a = \frac{F}{m} = \frac{9.6\times10^{-16}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = \mathbf{1.1\times10^{15}\ m/s^2\ upward}$$

## ELECTRIC POTENTIAL

(*College Physics* 9th ed. pages 548–558/10th ed. pages 558–568)

Electrical force is a conservative force, and as a result there is an electrical potential energy, $U$, associated with the electric force, $F$. The change in electrical potential energy between two points, $\Delta U$, is the work, $W$, done moving a charged particle between these two point in an electric field. In equation form, the work done is $\Delta U = W = Fd = Eq_0 d$.

We define the change in electric potential, $\Delta V$, as

$$\Delta V = \frac{\Delta U}{q_o} = \frac{W}{q_o}$$

The SI unit of electric potential is the volt, V, and is defined as a joule per coulomb $(1\ V = 1\ J/C)$.

The electric potential due to a point charge can also be calculated by

$$V = k\frac{q}{R}$$

Voltage, or electric potential, is a scalar quantity.

> **AP Tip**
>
> In lifting a mass in a gravitational field a distance, $h$, a force must be applied. The minimum force required is the weight, $mg$, of the mass. The force does work on the Earth-mass system. The work done is $W = mgh$.

### SAMPLE PROBLEM 9

How much work is done moving a small body with a charge of +25 $\mu$C from point $A$ to point $B$ through a potential difference of 40.0 V?

### SOLUTION TO PROBLEM 9

Solving $\Delta V = W/q_0$ for work done gives

$$W = \Delta V q_0 = (40.0\ \text{V})(25\times10^{-6}\ \text{C}) = \mathbf{0.001\ J}.$$

### SAMPLE PROBLEM 10

Find the electrical potential 2.0 cm from a point charge $q = -12\ \mu\text{C}$.

## SOLUTION TO PROBLEM 10

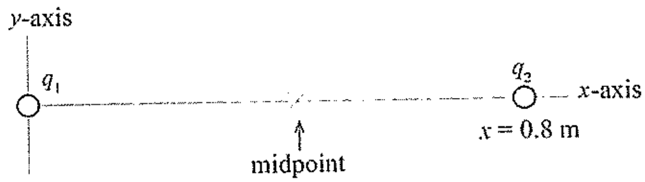
The electric potential is

$$V = k\frac{q}{R} = \frac{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(-12\times10^{-6}\ \mathrm{C}\right)}{(0.02\ \mathrm{m})} = \mathbf{-5.4\times10^{6}\ V}$$

Note: Since $V$ is not a vector quantity and cannot be represented by a directional vector, the negative sign must be included in the calculation of $V$.

## SAMPLE PROBLEM 11

A point charge of $q_1 = +2.0\ \mu\mathrm{C}$ is placed at the origin of a frame of reference, and a second point charge of $q_2 = -8.4\ \mu\mathrm{C}$ is placed at the position $x = 80.0$ cm as shown. Calculate the potential midway between these point charges.




## SOLUTION TO PROBLEM 11

The total electric potential of an arrangement of two or more charges by superposition is

$$V = k\sum\left(\frac{q_i}{r_i}\right) = \left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left[\left(\frac{2.0\times10^{-6}\ \mathrm{C}}{0.4\ \mathrm{m}}\right) + \left(\frac{-8.4\times10^{-6}\ \mathrm{C}}{0.4\ \mathrm{m}}\right)\right]$$

Simplifying the algebra, we have

$$V = \frac{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)}{0.4\ \mathrm{m}}\left[(2.0)+(-8.4)\right]\times10^{-6}\ \frac{\mathrm{C}}{\mathrm{m}} = \mathbf{-1.44\times10^{5}\ V}$$

Since $V$ is a scalar quantity, the potentials were added algebraically.

## SAMPLE PROBLEM 12

In air, a metal sphere of radius $R$ = 10.0 cm is given an electrical charge of $q = +100$ nC.

(a) What is the electrical potential at the surface of the sphere?

(b) What is the electrical potential at a point 20.0 cm from the surface of the sphere?

(c) Determine the maximum electrical field intensity for the sphere.

## SOLUTION TO PROBLEM 12

(a) The electric potential at the surface of the sphere is

$$V = k\frac{q}{R} = \frac{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(100\times10^{-9}\ \mathrm{C}\right)}{(0.10\ \mathrm{m})} = \mathbf{9\times10^{9}\ V}$$

(b) The electric potential at a point 20.0 cm from the surface of the sphere is

$$V = k\frac{q}{R} = \frac{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(100\times10^{-9}\ \mathrm{C}\right)}{(0.10\ \mathrm{m}+0.20\ \mathrm{m})} = \mathbf{3\times10^{3}\ V}$$

(c) The maximum electrical field intensity exists on the surface of the sphere. So

$$E = k\frac{q}{R^2} = \frac{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(100\times10^{-9}\mathrm{C}\right)}{(0.10\ \mathrm{m})^2} = \mathbf{9\times10^{4}\ N/C}$$

Notice that a relationship exists between $E$ and $V$.

$$V = \frac{kq}{R} \text{ and } E = \frac{kq}{R^2}$$

$$VR = kq = ER^2$$

Therefore

$$V = ER$$

So, reworking part (c), we then have

$$E = \frac{V}{R} = \frac{9\times10^{3}\ \mathrm{V}}{0.1\ \mathrm{m}} = \mathbf{9\times10^{4}\ V/m}$$

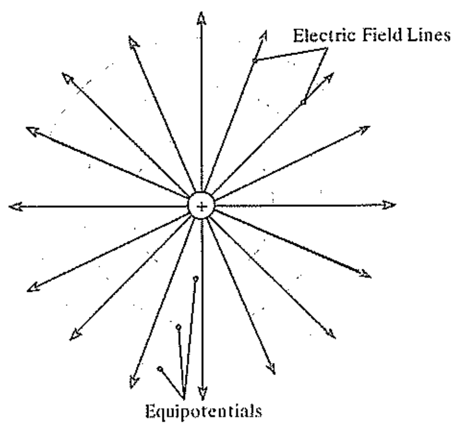
All electrical fields have two properties at every point in space:

1. The electrical field, $E$, which allows us to calculate the force acting on any charged particle placed at that point.
2. Electrical potential, $V$, which allows us to find the work done transporting a point charge to that position from a great distance away.

## EQUIPOTENTIAL SURFACES

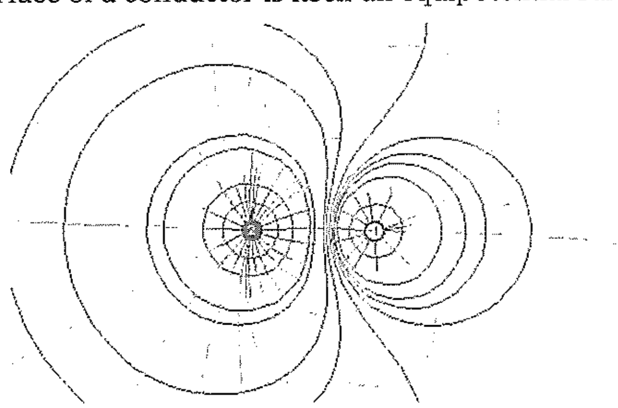
(*College Physics* 9th ed. pages 559–560/10th ed. pages 570–571)

A collection of points that all have the same electrical potential constitutes what is called an equipotential surface. The equipotential surfaces of a spherical surface are concentric spheres perpendicular to the electric field lines as shown. For a region in space where an electric field exists, the equipotential surfaces are always perpendicular to the electrical field lines.




The work done to move a charge, $q$, between two points on an equipotential surface is $W = q\Delta V = 0$ regardless of the path taken because every point on the surface is at the same electric potential.

The surface of a conductor is itself an equipotential surface.



The lines of force and the equipotential lines associated with a point charges $+2q$ and $-q$ in electrostatic attraction.

## DIELECTRIC STRENGTH

(*College Physics* 9th ed. pages 573–577/10th ed. pages 584–587)

There is a limit to the maximum electrostatic field that air will support. This limit is called the dielectric strength, and for air it is $3\times10^{6}$ V/m. When this value is exceeded, the air molecules in the field ionize, causing the charge creating the field to be reduced. The maximum electrical potential in a medium such as air is related to the maximum electrical field the air will support by

$$V_{max} = E_{max}R$$

### Sample Problem 13

Consider Example 11 again. In air, a metal sphere of radius $R = 10.0$ cm is given an electrical charge of $q = +100\ \text{nC}$. What is the electrical potential at the surface of the sphere?

### Solution to Problem 13

For a charged metallic sphere, the charge exists on the surface. The maximum electrical potential that can exist on the surface of the sphere is

$$V_{max} = E_{max}R = \left(3\times10^{6}\ \frac{\text{V}}{\text{m}}\right)(0.10\ \text{m}) = \mathbf{3\times10^{5}\ V}$$

The electrical potential at the surface of Example 11 is less than this value.

## Electrical Potential Energy

(*College Physics* 9th ed. pages 555–558/10th ed. pages 565–568)

The total energy of a charge in an electrical field is conserved. The electrical potential energy, $U$, of two point charges separated by distance $R$ is

$$U = k\frac{q_1 q_2}{R}$$

### Sample Problem 14

Two point charges, $q_A = +12\ \mu\text{C}$ and $q_B = -22\ \mu\text{C}$, are brought from infinity to a distance of 0.40 m of one another. How much work was done to assemble this system?

### Solution to Problem 14

First, find the electrical potential energy.

$$\begin{aligned} U &= k\frac{q_A q_B}{R} \\ &= \frac{\left(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+12\times10^{-6}\ \text{C}\right)\left(-22\times10^{-6}\ \text{C}\right)}{(0.40\ \text{m})} \\ &= \mathbf{-5.9\ J} \end{aligned}$$

Because the particles attract, their fields do $-5.9\ \text{J}$ work in assembling the system. To separate the particles to infinity, some outside agent would have to do $+5.9\ \text{J}$ of work.

Note: Since $U$ is not a vector quantity and cannot be represented by a directional vector, the negative sign must be included in the calculation of $U$.

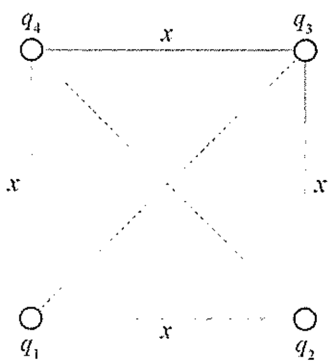
## SAMPLE PROBLEM 15

Four point charges, $q_1 = 2.5\ \mu\text{C}$, $q_2 = -1.5\ \mu\text{C}$, $q_3 = 7.5\ \mu\text{C}$ and $q_4 = -5.5\ \mu\text{C}$ are arranged in a square measuring $x = 0.35\ \text{m}$ on a side.

(a) How much work was done by outside forces to arrange the charges in the square?

(b) What is the electric potential $V$ at the center of the square?

(c) An electron initially at rest starts from a great distance away and moves toward the center of the square. What classical velocity will the electron have when it reaches the center?




## SOLUTION TO PROBLEM 15

(a) The point charges are assembled one charge at a time. To place $q_1$ requires zero work, $W_1 = 0$, since there are no other charges present. Positioning $q_2$ requires outside work $W_2$ that equals the electrical potential energy of $q_1$ and $q_2$. So,

$$W_2 = U = k\frac{q_1 q_2}{x} = \frac{k}{x}(q_1 q_2)$$

$$W_2 = \frac{\left(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.35\ \text{m}}\left(2.5\times10^{-6}\ \text{C}\right)\left(-1.5\times10^{-6}\ \text{C}\right)$$

$$W_2 = \frac{\left(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.35\ \text{m}}\left[(2.5)(-1.5)\right]\times10^{-6}\ \text{C} = \mathbf{-0.096\ J}$$

To place the third point charge, additional work is due to the two point charges already in place. A diagonal of the square forms a right triangle with sides $x$ and hypotenuse $x\sqrt{2}$. Each diagonal has a length $x\sqrt{2}$. So

$$W_3 = kq_3\left(\frac{q_1}{x_{13}} + \frac{q_2}{x_{23}}\right)$$

$$W_3 = kq_3\left(\frac{q_1}{x\sqrt{2}} + \frac{q_2}{x}\right)$$

$$W_3 = \left(9\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(7.5\times10^{-6}\ \mathrm{C}\right)\left[\left(\frac{2.5\times10^{-6}\ \mathrm{C}}{\sqrt{2}(0.35\ \mathrm{m})}\right)+\left(\frac{-1.5\times10^{-6}\ \mathrm{C}}{0.35\ \mathrm{m}}\right)\right]$$

Simplifying gives

$$W_3 = \left(9\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(7.5\times10^{-6}\ \mathrm{C}\right)\left[\left(\frac{2.5}{0.49}\right)+\left(\frac{-1.5}{0.35}\right)\right]\times10^{-6}\ \frac{\mathrm{C}}{\mathrm{m}} = \mathbf{0.055\ J}$$

Positioning point charge $q_4$ requires work of $W_4$.

$$W_4 = kq_4\left(\frac{q_1}{x_{14}} + \frac{q_2}{x_{24}} + \frac{q_3}{x_{34}}\right)$$

$$W_4 = kq_4\left(\frac{q_1}{x} + \frac{q_2}{x\sqrt{2}} + \frac{q_3}{x}\right)$$

$$W_4 = \left(9\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(-5.5\times10^{-6}\ \mathrm{C}\right)\left[\left(\frac{2.5\times10^{-6}\ \mathrm{C}}{0.35\ \mathrm{m}}\right)+\left(\frac{-1.5\times10^{-6}\ \mathrm{C}}{(\sqrt{2})(0.35\ \mathrm{m})}\right)+\frac{(7.5\times10^{-6}\ \mathrm{C})}{0.35\ \mathrm{m}}\right]$$

Simplifying yields

$$W_4 = \left(9\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(-5.5\times10^{-6}\ \mathrm{C}\right)\left[\left(\frac{2.5}{0.35}\right)+\left(\frac{-1.5}{0.49}\right)+\frac{(7.5)}{0.35}\right]\times10^{-6}\ \frac{\mathrm{C}}{\mathrm{m}} = \mathbf{-1.26\ J}$$

Therefore, the total work is $\Sigma W = W_1 + W_2 + W_3 + W_4$.

$$\Sigma W = 0 + (-0.096\ \mathrm{J}) + 0.055\ \mathrm{J} + (-1.26\ \mathrm{J}) = \mathbf{-1.30\ J}$$

(b) With all of the point charges in place, the potential at the center is

$$V = V_1 + V_2 + V_3 + V_4 = k\left(\frac{q_1+q_2+q_3+q_4}{(0.5x)(\sqrt{2})}\right)$$

$$V = \left(9\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{2.5-1.5+7.5-5.5}{(0.175)(\sqrt{2})}\right)\times10^{-6}\ \frac{\mathrm{C}}{\mathrm{m}}$$

$$= \mathbf{1.09\times10^5\ V}$$

(c) The electron must lose electrical potential energy $\Delta U = Ve$.

$$\Delta U = \left(1.09\times10^5\mathrm{V}\right)\left(-1.60\times10^{-19}\ \mathrm{C}\right) = \mathbf{-1.75\times10^{-14}\ J}$$

This electrical potential energy becomes kinetic energy at the center of the square and $K = 1.75\times10^{-14}$ J. Therefore $K = \frac{1}{2}mv^2$ and

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1.74\times10^{-14}\ \mathrm{V}\right)}{9.11\times10^{-31}\ \mathrm{kg}}} = \mathbf{1.96\times10^8\ \frac{m}{s}}$$

## THE PARALLEL-PLATE CAPACITOR

(*College Physics* 9th ed. pages 563–565/10th ed. pages 573–576)

One of the most useful electrical devices in many aspects of electricity is the parallel-plate capacitor. The plates are metal and are separated by a plate gap, *d*. Connecting the capacitor to a battery charges the plates. Think of a battery as an electron pump. It removes electrons from what becomes the positive plate and places an equal number of electrons on what becomes the negative plate (charging a capacitor is done by electrostatic repulsion). It takes a brief period of time to charge the capacitor, and then the battery can be removed. The magnitude of the charges on the plates are equal, $|q_{plate\ 1}| = |q_{plate\ 2}|$. The amount of charge on each plate is proportional to the potential difference, *V*, across the plates

$$C = \frac{q}{V}$$

We call the constant of proportionality, *C*, the capacitance of the capacitor.

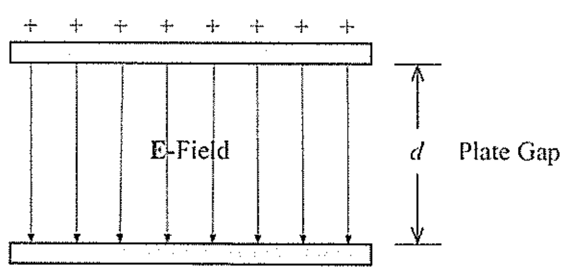
The SI unit of capacitance is the farad, F, and is defined as $1\ \text{F} = 1\ \text{C/V}$. The farad is a very large unit. The usual capacitance of a capacitor is on the order of several microfarads, $\mu\text{F}$, to several picofarads $\text{pF}$. In short

$$1\ \mu\text{F} = 1 \times 10^{-6}\ \text{F}$$

$$1\ \text{nF} = 1 \times 10^{-9}\ \text{F}$$

$$1\ \text{pF} = 1 \times 10^{-12}\ \text{F}$$

The upper plate in the diagram shown below has been charged positive. The field always points from positive charge to negative. Ignoring the edges of the plates, the electrical field between the plates is uniform.




For a parallel-plate capacitor, the relationship between the electrical field, *E*, between the plates, the potential difference across the plates and the plate gap is

$$V = Ed$$

## SAMPLE PROBLEM 16

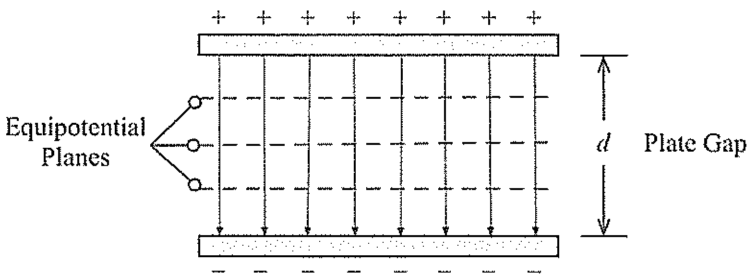
A parallel plate capacitor has a potential difference of 100 V across its plates and a plate gap of 2.0 mm. What electric field, $E$, exists between the plates?

## SOLUTION TO PROBLEM 16

$$E = \frac{V}{d} = \frac{100\text{ V}}{2\times10^{-3}\text{m}} = \mathbf{5\times10^4\ V/m}$$

The student should prove that prove that $1\ \text{N/C} = 1\ \text{V/m}$.

The equipotential surfaces between the plates of a parallel capacitor are planes that are perpendicular to the field lines.




The capacitance of a parallel-plate capacitor can be calculated by using

$$C = \varepsilon_o \frac{A}{d}$$

where $\varepsilon_o$ is the permittivity constant of free space and $A$ is the area of one of the plates. It gives us information on how well an E-field is set up in space. In SI units, $\varepsilon_o = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

## SAMPLE PROBLEM 17

A set of capacitor plates each measures 10.0 cm by 12.0 cm, and they have a plate gap of 3.0 mm. Calculate the capacitance.

## SOLUTION TO PROBLEM 17

The area of one of the plates is

$$120\ \text{cm}^2 \times \frac{1\ \text{m}^2}{1\times10^4\text{cm}^2} = 120\times10^{-4}\text{m}^2$$

Therefore

$$C = \varepsilon_o \frac{A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(120\times10^{-4}\text{m}^2)}{3\times10^{-3}\text{m}} = \mathbf{35\ pF}$$

## DIELECTRICS

(*College Physics* 9th ed. pages 573–578/10th ed. pages 584–587)

When an insulating material called a dielectric is inserted between the plates of a capacitor, the capacitance increases. The dielectric is represented by a dimensionless quantity called the dielectric constant, $\kappa$. With a dielectric inserted between the plates, the capacitance becomes

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

### SAMPLE PROBLEM 18

A parallel-plate capacitor of plate gap $d = 2.0$ mm has a capacitance $C_0 = 3.0\ \mu\text{F}$. A battery is used to charge the plates with a potential difference $V_0 = 600$ V. After the charging process, the battery is removed.

(a) What is the potential difference across the capacitor when a 2.0 mm thick slab of a dielectric of $\kappa = 7.5$ is sandwiched between the plates?

(b) What is the new capacitance?

(c) What is the permittivity of the dielectric?

### SOLUTION TO PROBLEM 18

Proportionally, the dielectric constant is $\kappa = V_0/V$ and

(a) $V = V_0/\kappa = 600\ \text{V}/7.5 = \mathbf{80\ V}$

(b) $C = \kappa C_0 = 7.5(3.0\times10^{-6}\ \text{F}) = \mathbf{22.5\ \mu F}$

(c) $\varepsilon = \kappa\varepsilon_0 = 7.5(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = \mathbf{6.64\times10^{-11}\ C^2/N\cdot m^2}$

## ELECTRICAL ENERGY STORED IN A CAPACITOR

(*College Physics* 9th ed. pages 571–573/10th ed. pages 582–584)

Parallel-plate capacitors not only produce uniform electrical fields between their plates and store charge on the plates; they also store electrical energy. The energy stored in the field between the plates may be calculated by

$$U = \frac{1}{2}CV^2$$

or $$U = \frac{1}{2}qV$$

or $$U = \frac{1}{2}\frac{q^2}{C}$$

### Sample Problem 19

A parallel-plate capacitor with a plate gap of 0.6 mm has a capacitance of 6.0 $\mu$F. A battery charges the plates with a potential difference of 500 V and is then disconnected.

(a) Calculate the energy stored in the capacitor.

(b) What charge exists on the plates?

### Solution to Problem 19

(a) Because the capacitance and potential difference are given, we will use $U = \frac{1}{2}CV^2$ to find the energy stored. Therefore

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(6.0\times10^{-6}\,\text{F}\right)\left(500\ \text{V}\right)^2 = \mathbf{0.75\ J}$$

(b) The charge is then

$$q = CV = \left(6.0\times10^{-6}\text{F}\right)\left(500\ \text{V}\right) = \mathbf{3\times10^{-3}\ C}$$

## Combinations of Capacitors

(*College Physics* 9th ed. pages 565–571/10th ed. pages 576–582)

Electrical circuits frequently contain two or more capacitors grouped together to serve a particular function. In considering the effect of such groupings, it is convenient to use a circuit diagram. In such diagrams, electrical components are represented by symbols. The symbol for a battery is a set of unequal parallel lines. The high-potential terminal (+ terminal) of a battery is represented by the longer line. The capacitor is diagramed as a set of equal-length parallel lines. Wires are shown as lines that connect components. Electrical circuit diagrams are also called electrical schematics.

When capacitors are arranged in parallel, their combined capacitance is the arithmetic sum of the individual capacitors.

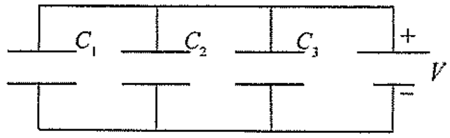
$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

### Sample Problem 20

Three capacitors, $C_1 = 0.5\mu$F, $C_2 = 0.3\ \mu$F and $C_3 = 0.2\ \mu$F are arranged in parallel. Calculate the equivalent capacitance.

## SOLUTION TO PROBLEM 20

We start with a circuit diagram and then substitute values.




$$C_{eq} = C_1 + C_2 + C_3 = (0.5 + 0.3 + 0.2)\,\mu\text{F} = \mathbf{1.0}\ \mu\text{F}$$

When capacitors are arranged in series, the reciprocal of the combined capacitance is the sum of the reciprocals of the capacitors.

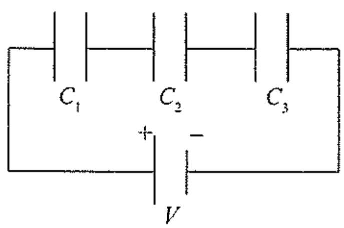
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

## SAMPLE PROBLEM 21

Three capacitors, $C_1 = 0.5\mu\text{F}$, $C_2 = 0.3\ \mu\text{F}$ and $C_3 = 0.2\ \mu\text{F}$, are arranged in series. Calculate the equivalent capacitance, $C_{eq}$.

## SOLUTION TO PROBLEM 21

Again we start with a circuit diagram and substitute




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{0.5\ \mu\text{F}} + \frac{1}{0.3\ \mu\text{F}} + \frac{1}{0.2\ \mu\text{F}}$$

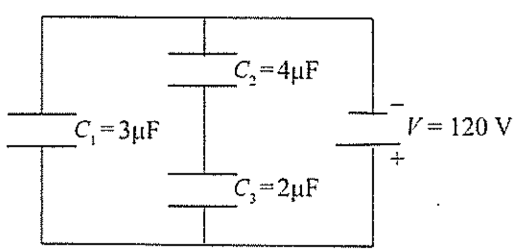
$$C_{eq} = \mathbf{0.097}\ \mu\text{F}$$

## SAMPLE PROBLEM 22

(a) Calculate the total capacitance of the circuit shown below.

(b) Find the charge on each capacitor in the circuit.

(c) Calculate the voltage drop across capacitor $C_2$.



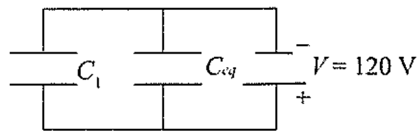
## SOLUTION TO PROBLEM 22

(a) Capacitors $C_2$ and $C_3$ are in series with one another and their equivalent capacitance is

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{4\ \mu\text{F}}+\frac{1}{2\ \mu\text{F}}$$

$$C_{eq}=\mathbf{1.33\ \mu F}$$

The circuit is now reduced as shown below.




The two remaining capacitances are arranged in parallel and $C=C_1+C_{eq}=3\ \mu\text{F}+1.33\ \mu\text{F}=\mathbf{4.33\ \mu F}$.

(b) The total charge in the capacitors is $q=CV=(4.33\ \mu\text{F})(120\ \text{V})=$ $520\ \mu\text{C}$. The charge, $q_1$, on $C_1$ is $q_1=C_1V=(3\ \mu\text{F})(120\ \text{V})=$ $\mathbf{360\ \mu C}$ and the remaining charge is $q-q_1=520\ \mu\text{C}-360\ \mu\text{C}=$ $160\ \mu\text{C}$. The $160\ \mu\text{C}$ must be deposited on capacitors $C_2$ and $C_3$. So, $q_2=q_3=\mathbf{160\ \mu C}$.

(c) The voltage drop is $V=q_2/C_2=\mathbf{40\ V}$.

## ELECTROSTATICS: STUDENT OBJECTIVES FOR THE AP EXAM

- You should know that there are two fundamental charges.
- You should understand Coulomb's law for the fundamental electrostatic force between point charges and that the electrical fields produced by these charges are vector quantities with magnitude and direction.
- You should know how to use the equations, identify the charges, and sketch the directions for the forces and field in solving problems that involve these quantities.
- You should know that potential difference and electrical potential, closely related concepts, are scalar quantities.
- You should know how to use the equations, identify the charges, and solve problems involving potential difference and electrical potential.
- You should know that the equations of potential difference and electrical potential can be used to solve conservation of energy and work–energy theorem problems.
- You should know the relationship between electric field lines and equipotential lines and be able to sketch them.
- You should know that the electron volt, eV, is a unit of work and energy.